Electrical Engineering
- Charges -

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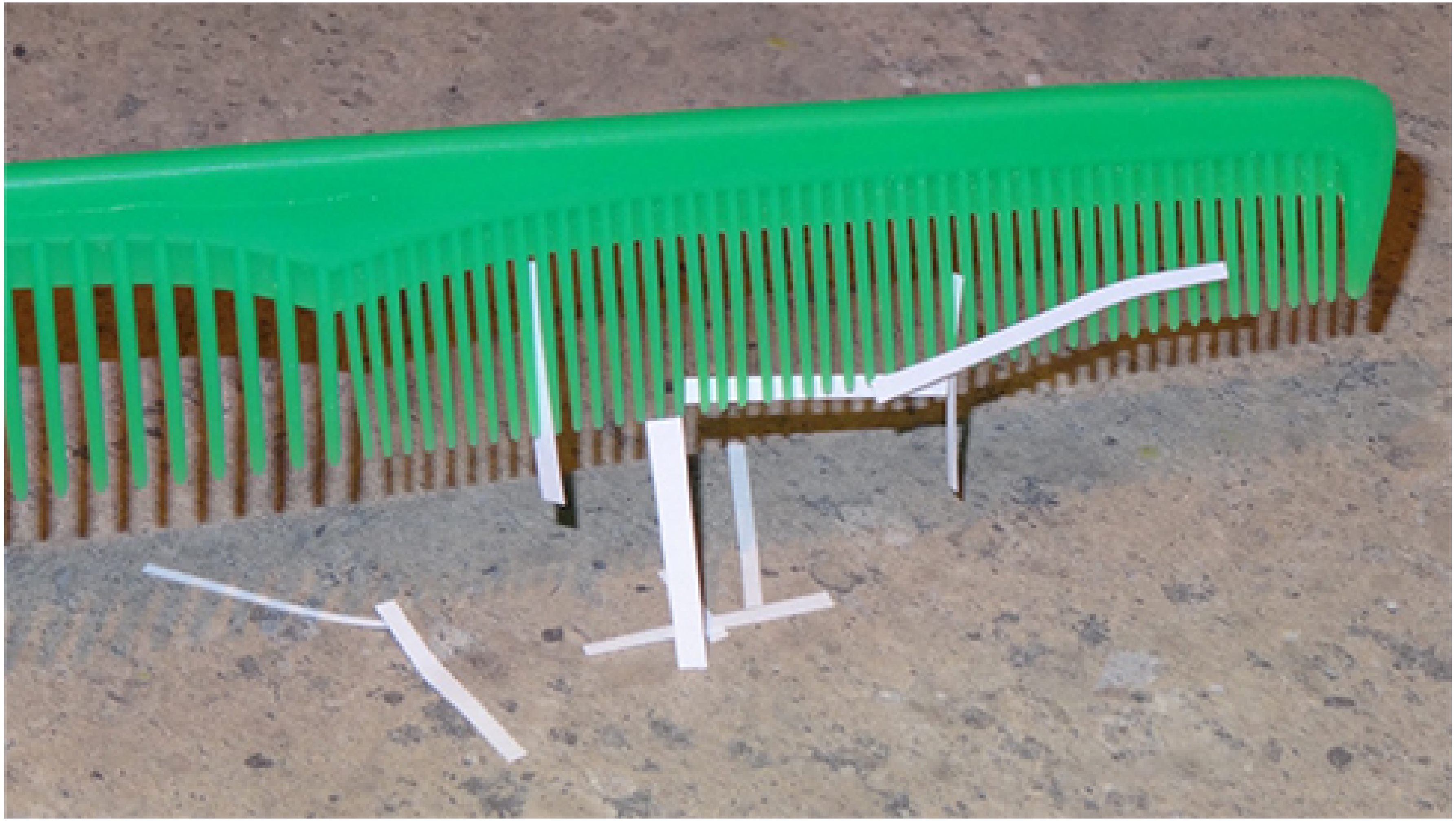
Environment and Energy Program, HSRW

Electric Charges and Fields

Source: University Physics II - Thermodynamics, Electricity, and Magnetism (OpenStax)

5.2: Electric Charge

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Charges can be separated, particularly if one material has a greater affinity for electrons than another.

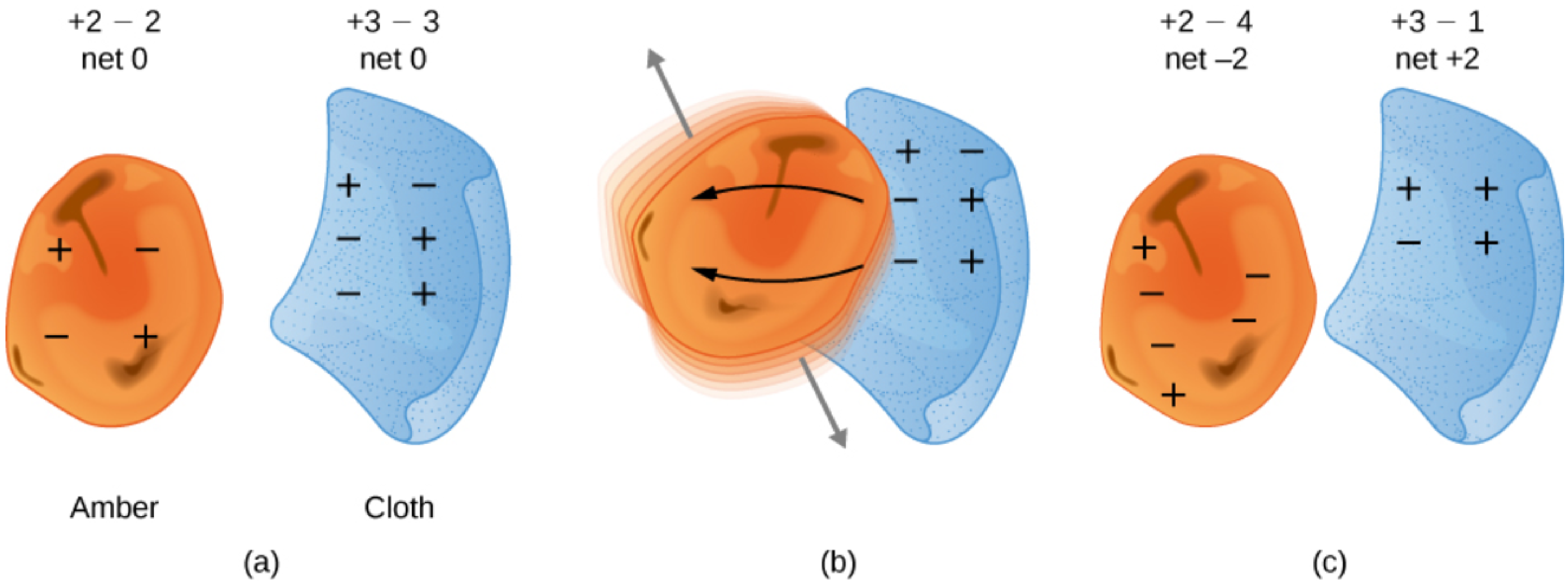
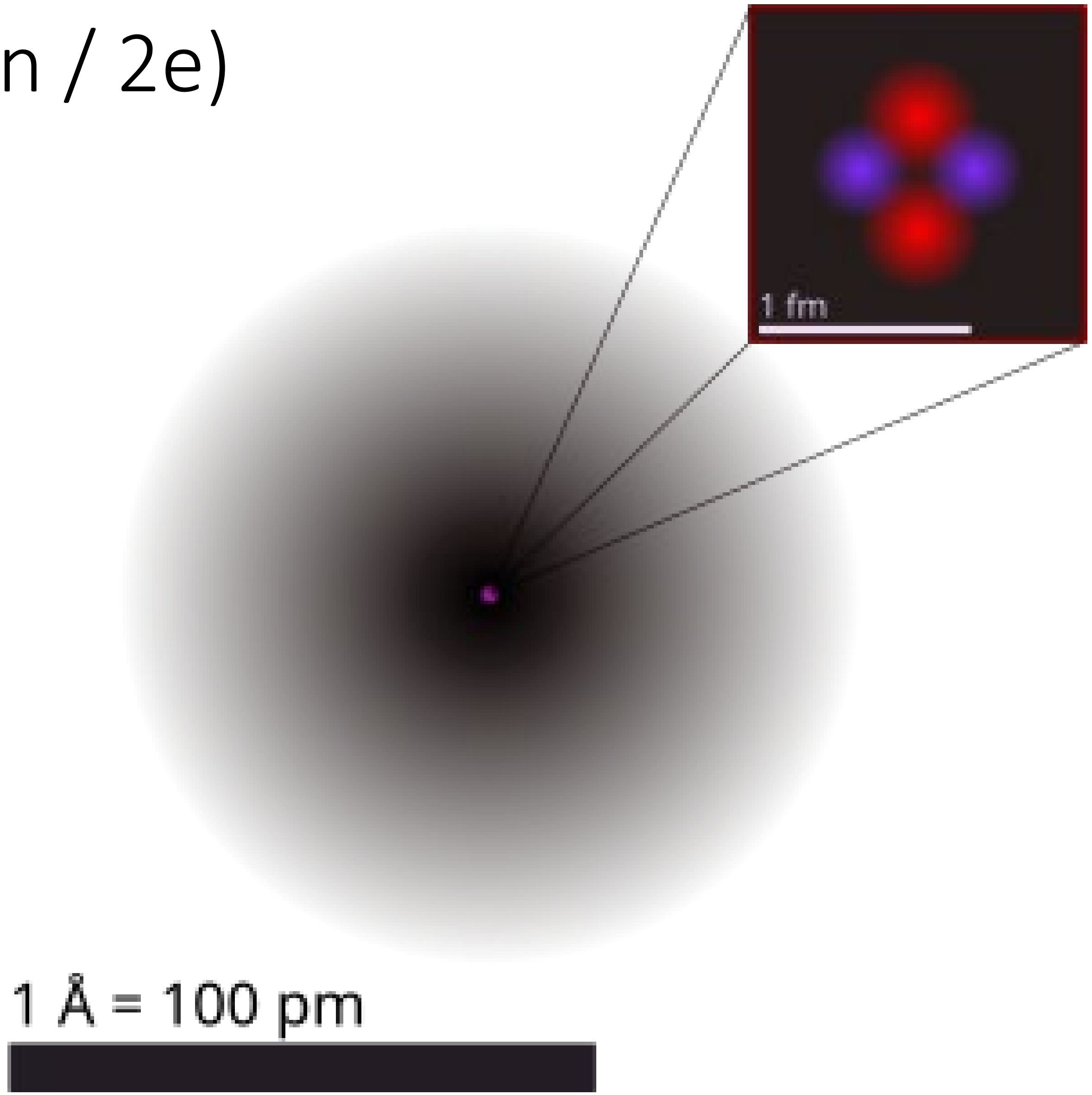


Figure 5.2.4: When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

Helium-4 Atom (2p 2n / 2e)

- Grey: Electron cloud, probability distribution of the 2 Electrons
- Red: 2 Protons
- Blue: 2 Neutron



https://en.wikipedia.org/wiki/Atom

Source of Charges: Structure of Atoms – ^{12}C

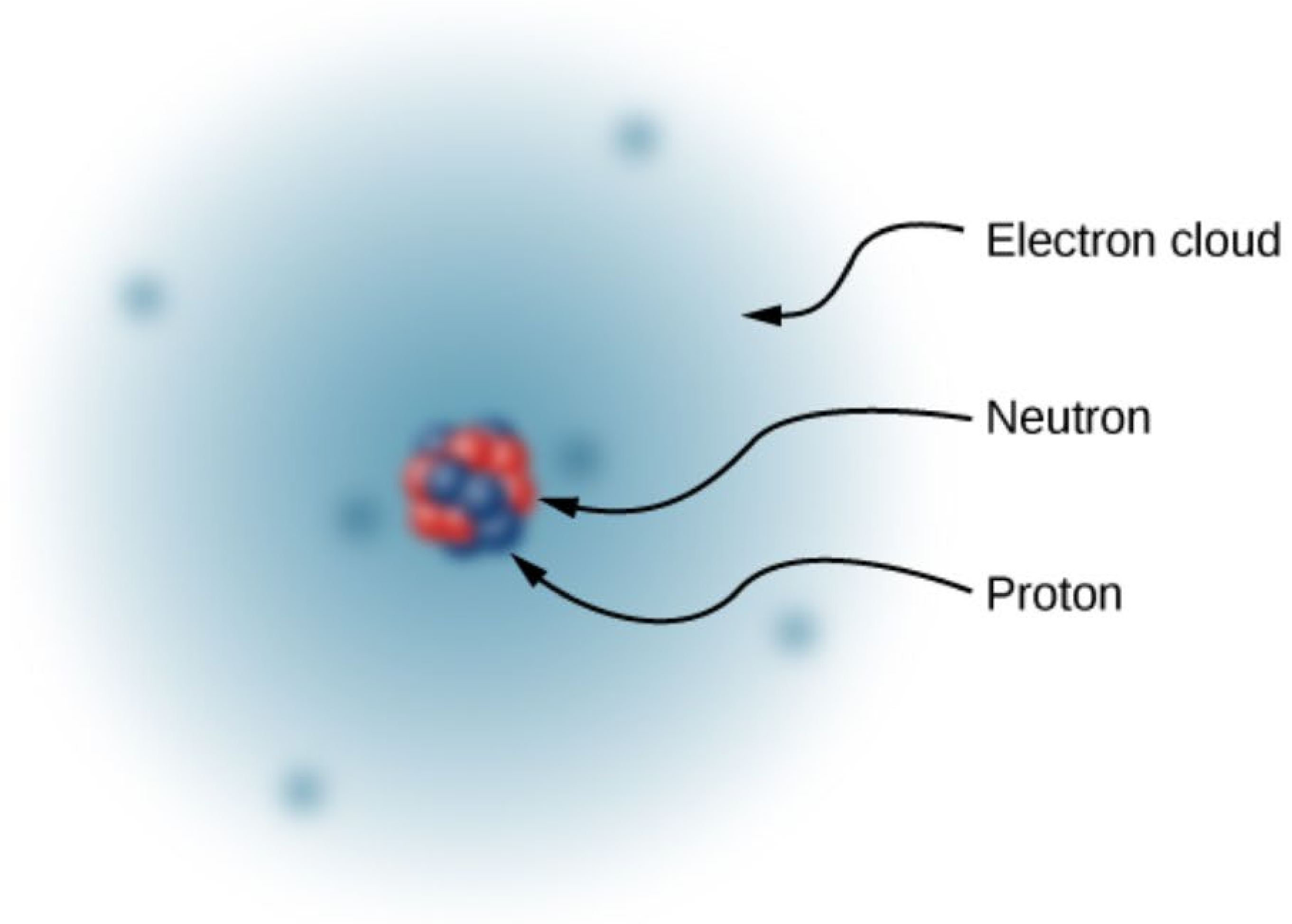
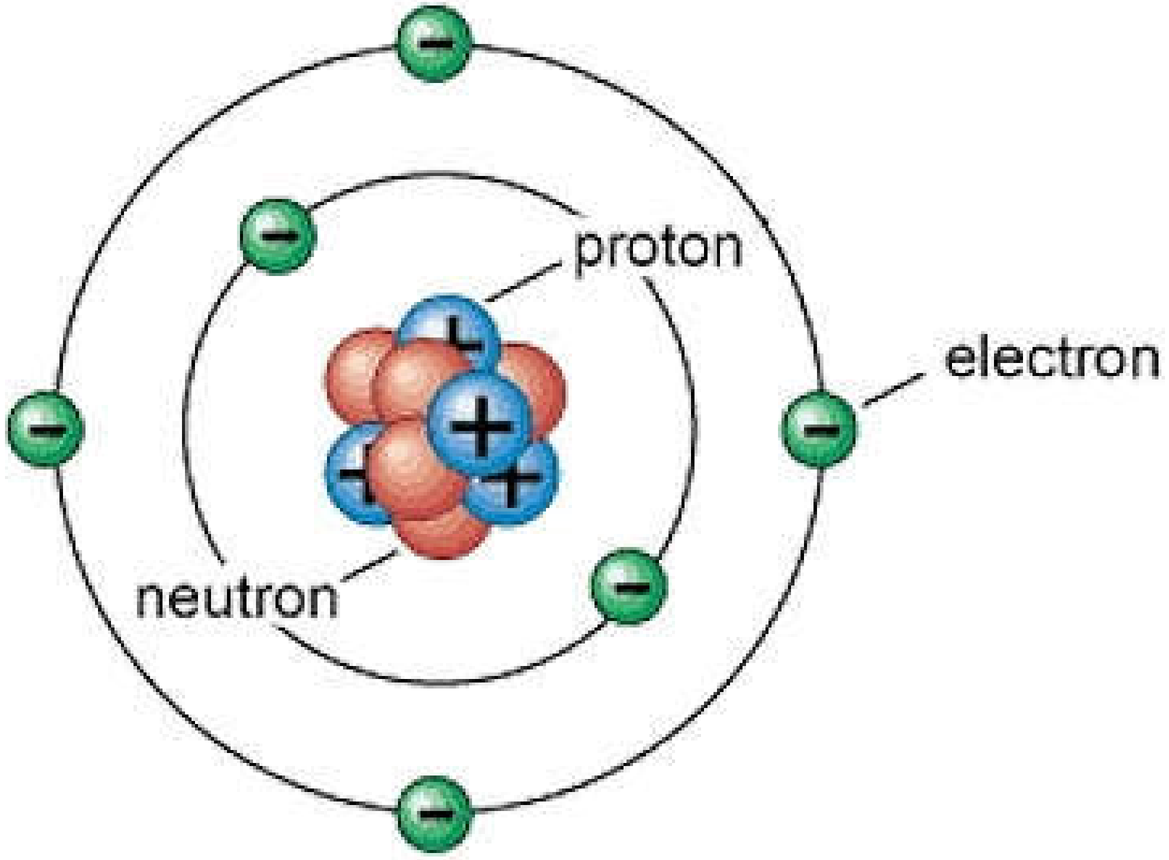


Figure 5.2.6: The nucleus of a carbon atom is composed of six protons and six neutrons. As in hydrogen, the surrounding six electrons do not have definite locations and so can be considered to be a sort of cloud surrounding the nucleus.

Simplifying Electron Orbital Model (inspired by Bohr)

- ^{12}C (Carbon-12)
- 6p, 6n
- 6e



https://www.researchgate.net/publication/362365182_Electron_theory_on_conductor_insulator_and_semiconductor/figures?lo=1

Charges exert forces on other charges. Interaction!

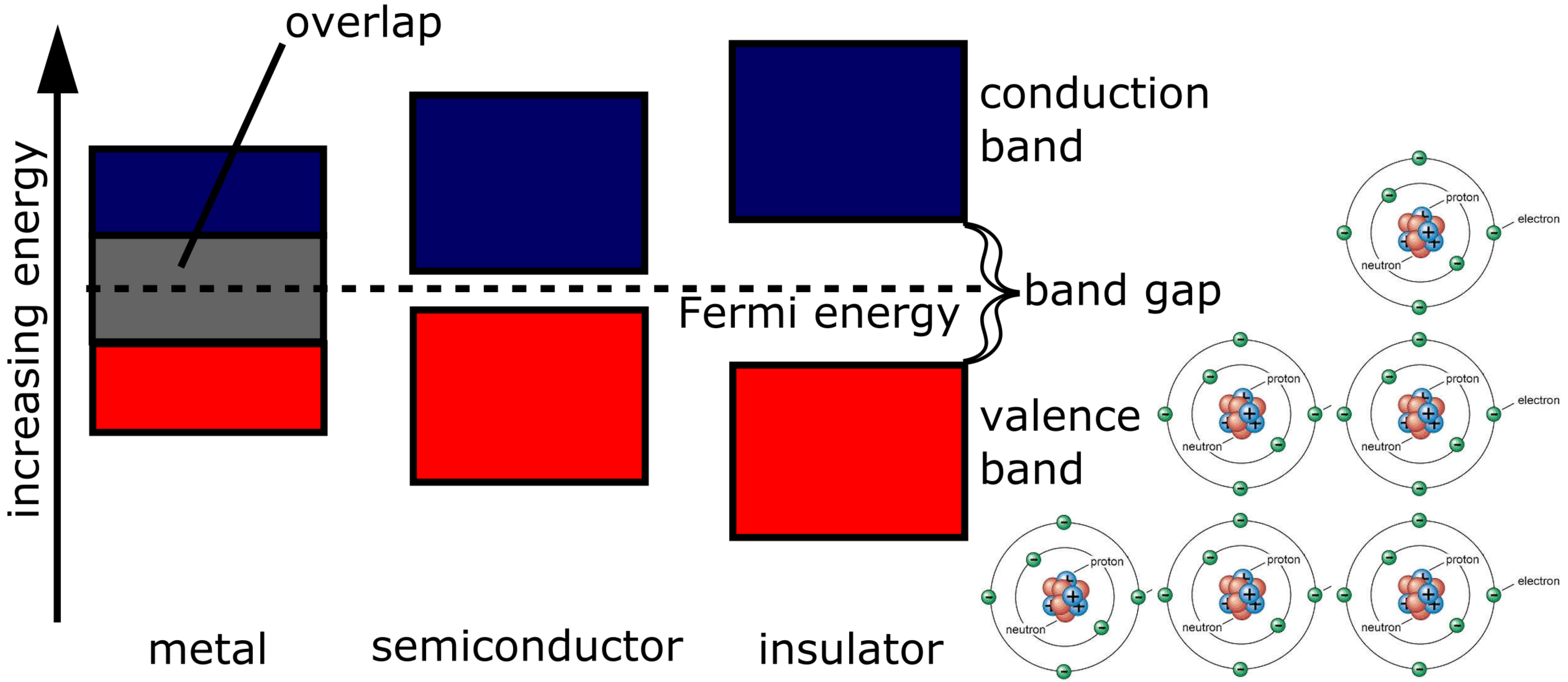
- The force acts **without physical contact** between the two objects.
- The force can be either **attractive or repulsive**: If two interacting objects carry the same sign of charge, the force is repulsive; if the charges are of opposite sign, the force is attractive. These interactions are referred to as **electrostatic repulsion and electrostatic attraction**, respectively.
- **Not all objects** are affected by this force.
- The **magnitude of the force decreases (rapidly)** with increasing separation distance between the objects.

Properties of Electric Charges

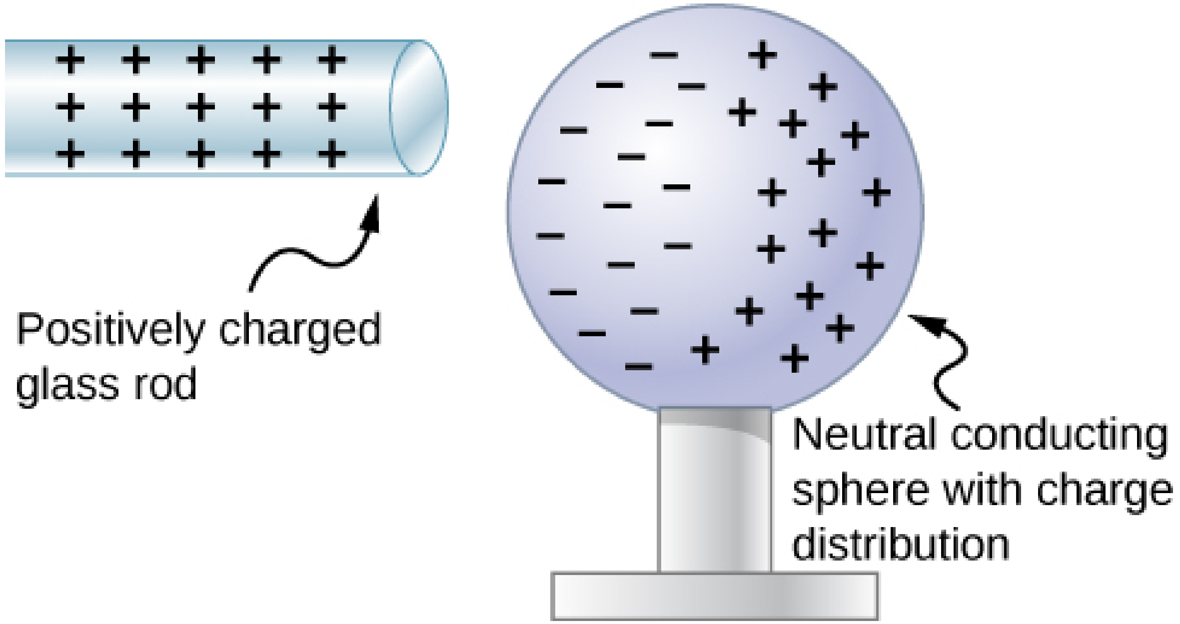
- **Charge is quantized**. An electric charge comes in discrete amountsIn the SI system, this **smallest amount** is $e \equiv 1.602\times10^{-19}$ C
- No free particle can have less charge than this, and, therefore, the charge on any object — the charge on all objects — **must be an integer multiple of this amount**. All macroscopic, charged objects have charge because electrons have either been added or taken away from them, resulting in a net charge.
- The magnitude of the charge is independent of the type. The smallest possible positive charge is $+1.602\times10^{-19}$ C (Coulomb) and the smallest possible negative charge is -1.602×10^{-19} C. These values are exactly equal.
- **Charge is conserved**. Charge can neither be created nor destroyed; it can only be transferred from place to place, from one object to another. Frequently, we speak of two charges “canceling”; this is verbal shorthand. It means that if two objects that have equal and opposite charges are physically close to each other, then the (oppositely directed) forces they apply on some other charged object cancel, for a net force of zero.

5.3: Conductors, Insulators, and Charging by Induction

Conductor, Semiconductor, Insulator

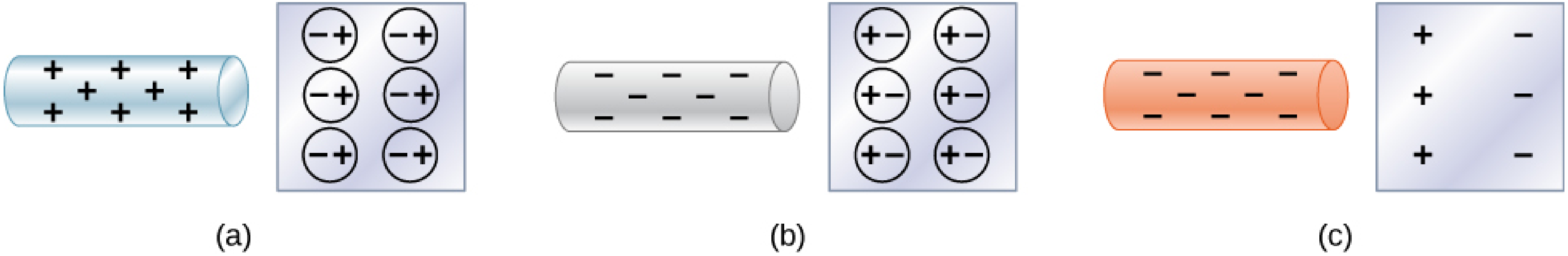


Induced Polarization



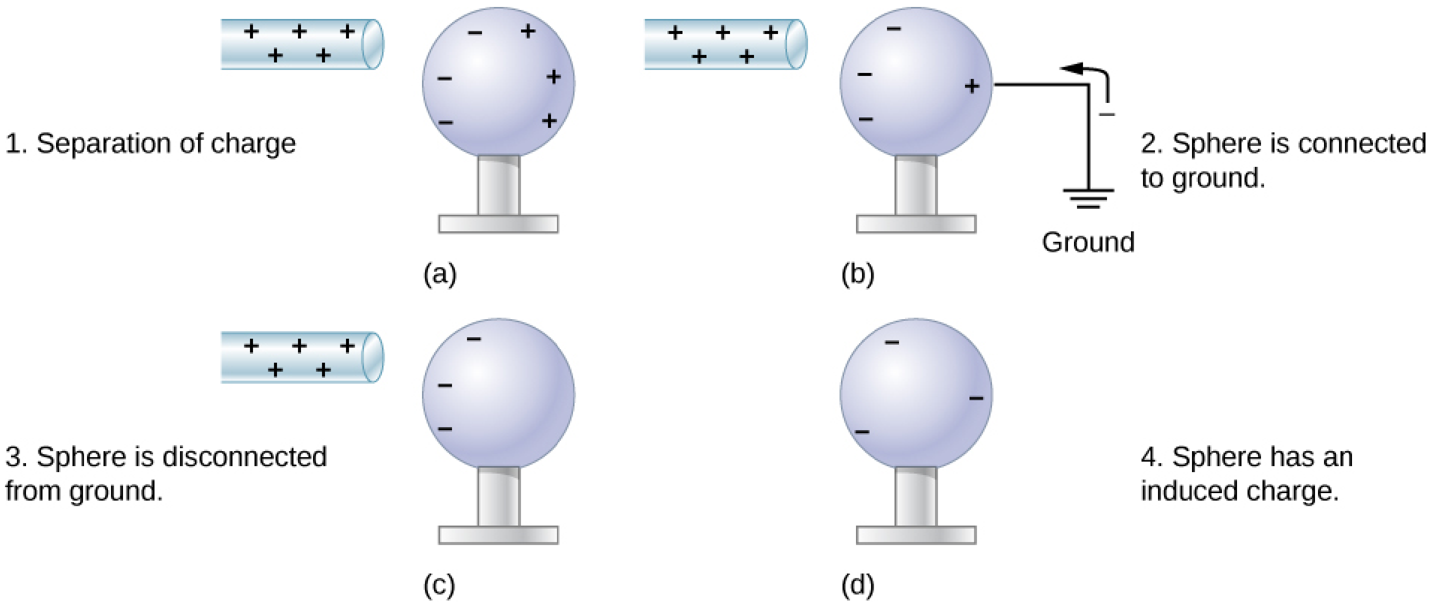
A positively charged glass rod is brought near the left side of the conducting sphere, attracting negative charge and leaving the other side of the sphere positively charged.

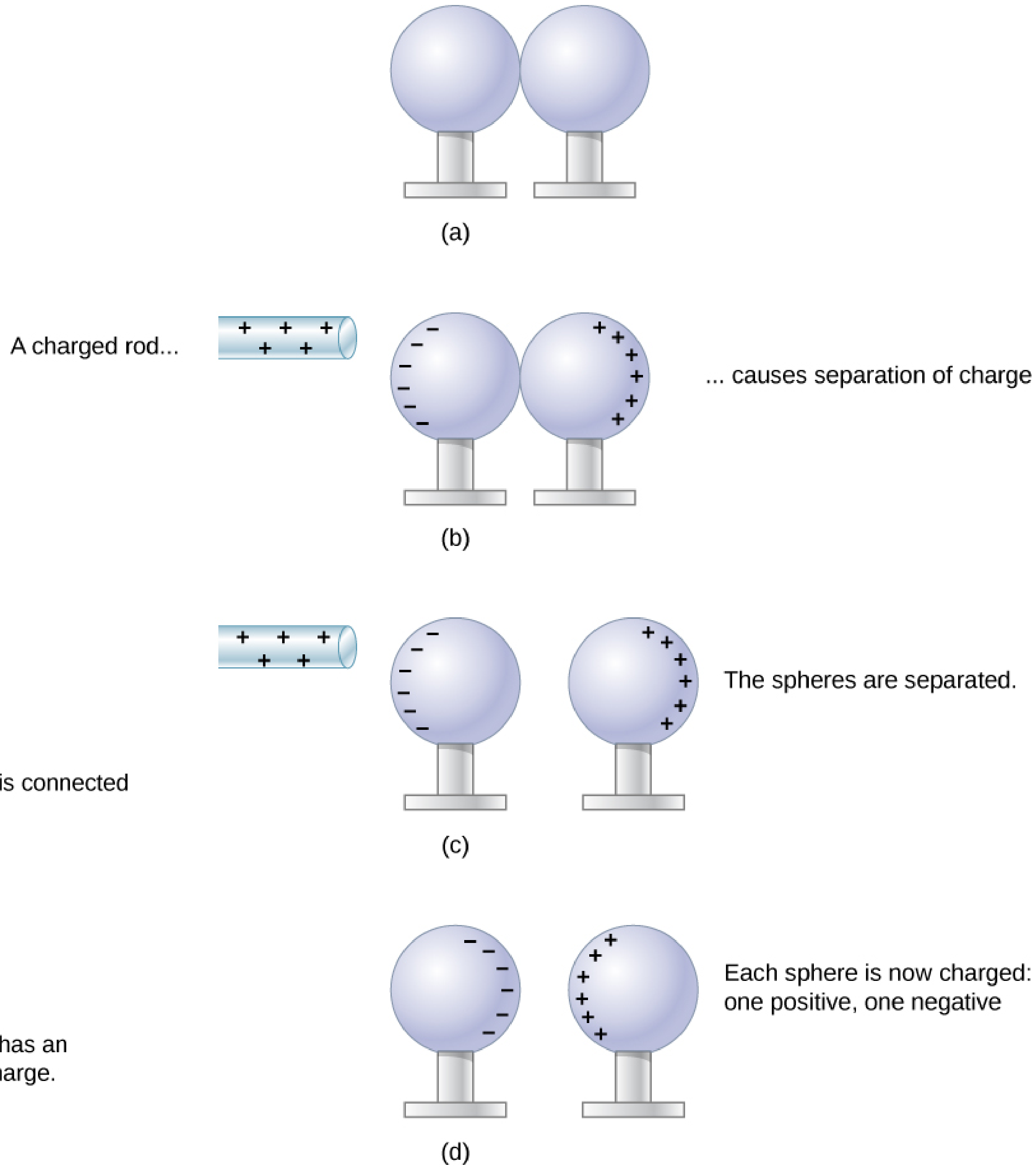
Charging by Induction



(a) A positive object brought near a neutral insulator polarizes its molecules. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Charging by induction
(Electrostatic influence)





Coulomb's Law, Coulomb Force, Electric Force

If two objects each have electric charge then they exert an electric force on each other.

$$|\mathbf{F}_{12}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r_{12}^2}$$

Permittivity of free space (vacuum):

$$\epsilon_0 = 8.85 \times 10^{-12}\frac{C^2}{N\cdot m^2}$$

Example 5.4.1: The Force on the Electron in Hydrogen

A hydrogen atom consists of a single proton and a single electron. The proton has a charge of $+e$ and the electron has $-e$. In the "ground state" of the atom, the electron orbits the proton at most probable distance of $5.29 \times 10^{-11} m$ (Figure 5.4.2). Calculate the electric force on the electron due to the proton.

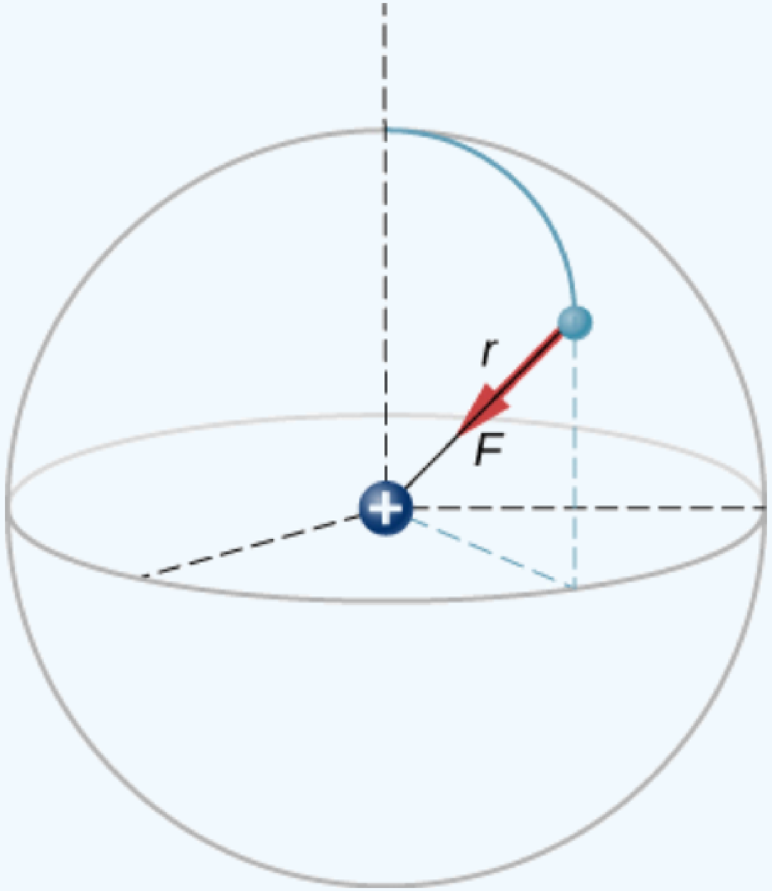
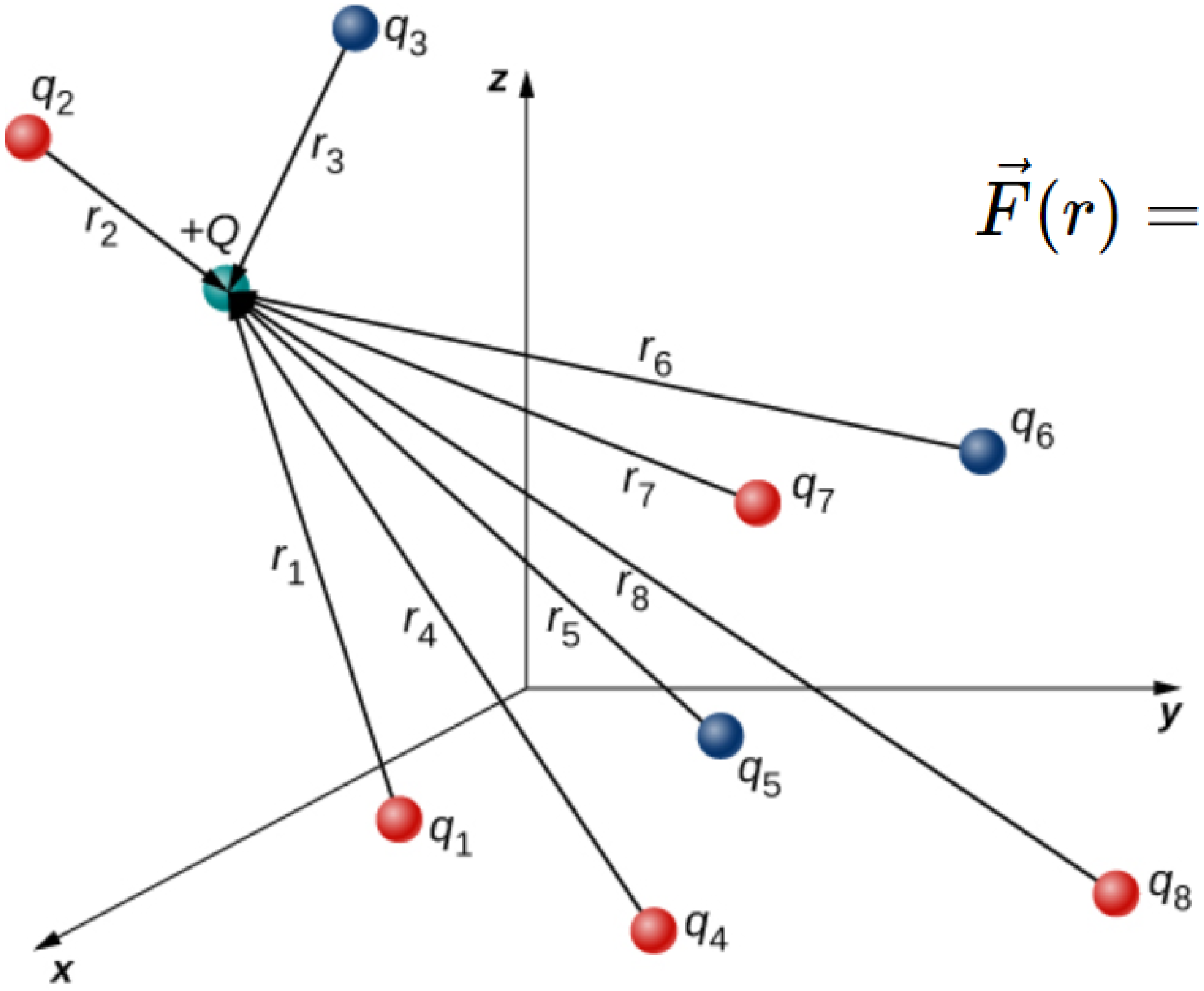


Figure 5.4.2: A schematic depiction of a hydrogen atom, showing the force on the electron. This depiction is only to enable us to calculate the force; the hydrogen atom does not really look like this.

Multiple Source Charges: Principle of Superposition



$$\vec{F}(r) = \frac{1}{4\pi\epsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i^2$$

Figure 5.4.3: The eight source charges each apply a force on the single test charge **Q**. Each force can be calculated independently of the other seven forces. This is the essence of the superposition principle.

✔ Example 5.4.2: The Net Force from Two Source Charges

Three different, small charged objects are placed as shown in Figure 5.4.2. The charges q_1 and q_3 are fixed in place; q_2 is free to move. Given $q_1 = 2e$, $q_2 = -3e$, and $q_3 = -5e$, and that $d = 2.0 \times 10^{-7} m$, what is the net force on the middle charge q_2?

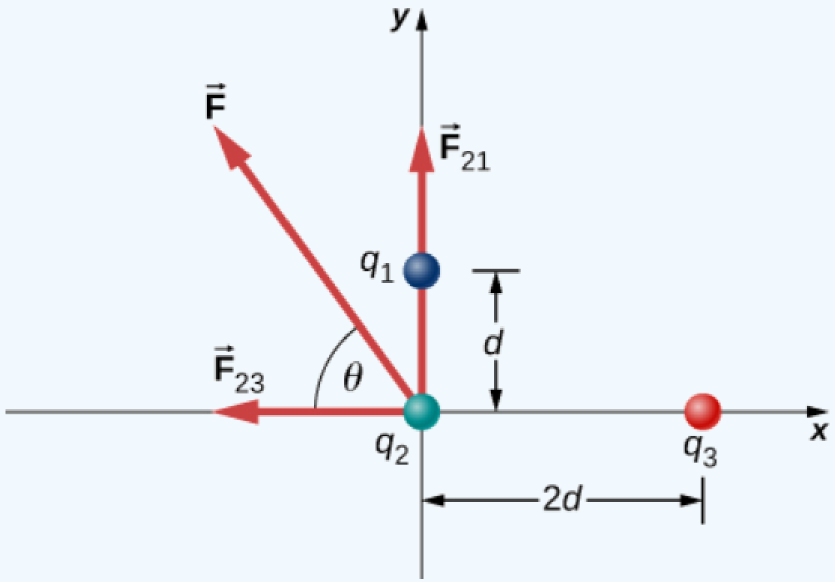


Figure 5.4.4: Source charges q_1 and q_3 each apply a force on q_2.

Strategy

We use Coulomb's law again. The way the question is phrased indicates that q_2 is our test charge, so that q_1 and q_3 are source charges. The principle of superposition says that the force on q_2 from each of the other charges is unaffected by the presence of the other charge. Therefore, we write down the force on q_2 from each and add them together as vectors.

✔ Example 5.4.2: The Net Force from Two Source Charges

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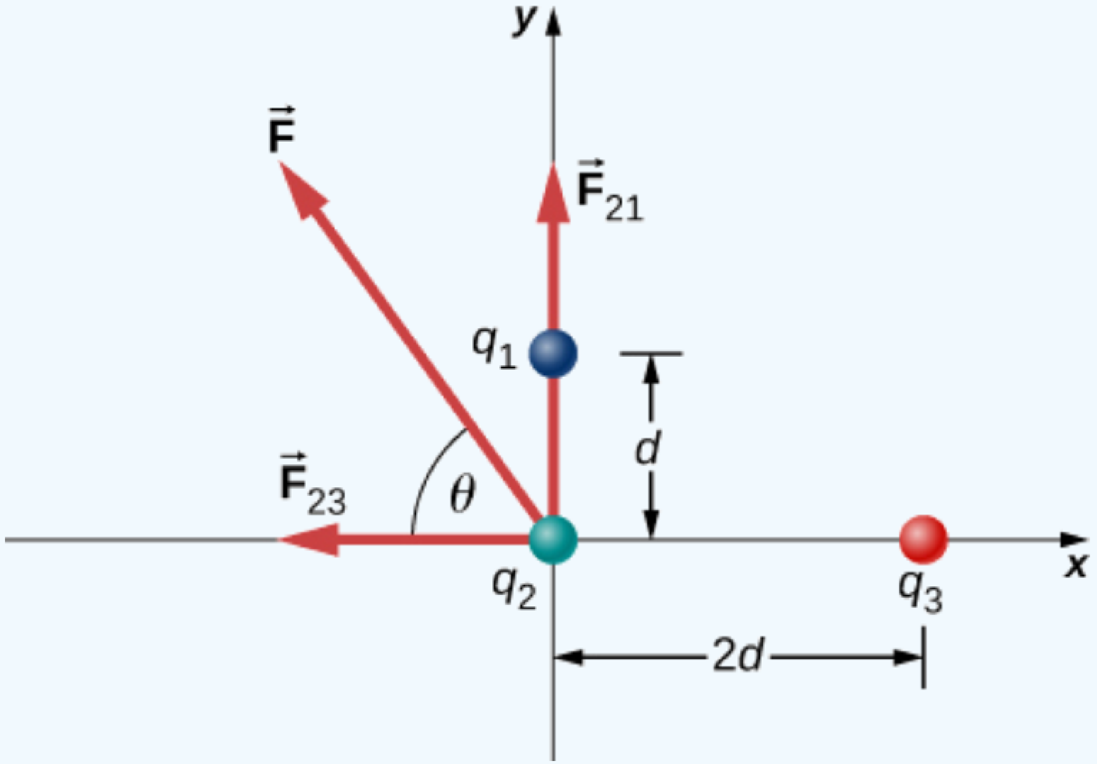


Figure 5.4.4: Source charges q_1 and q_3 each apply a force on q_2.

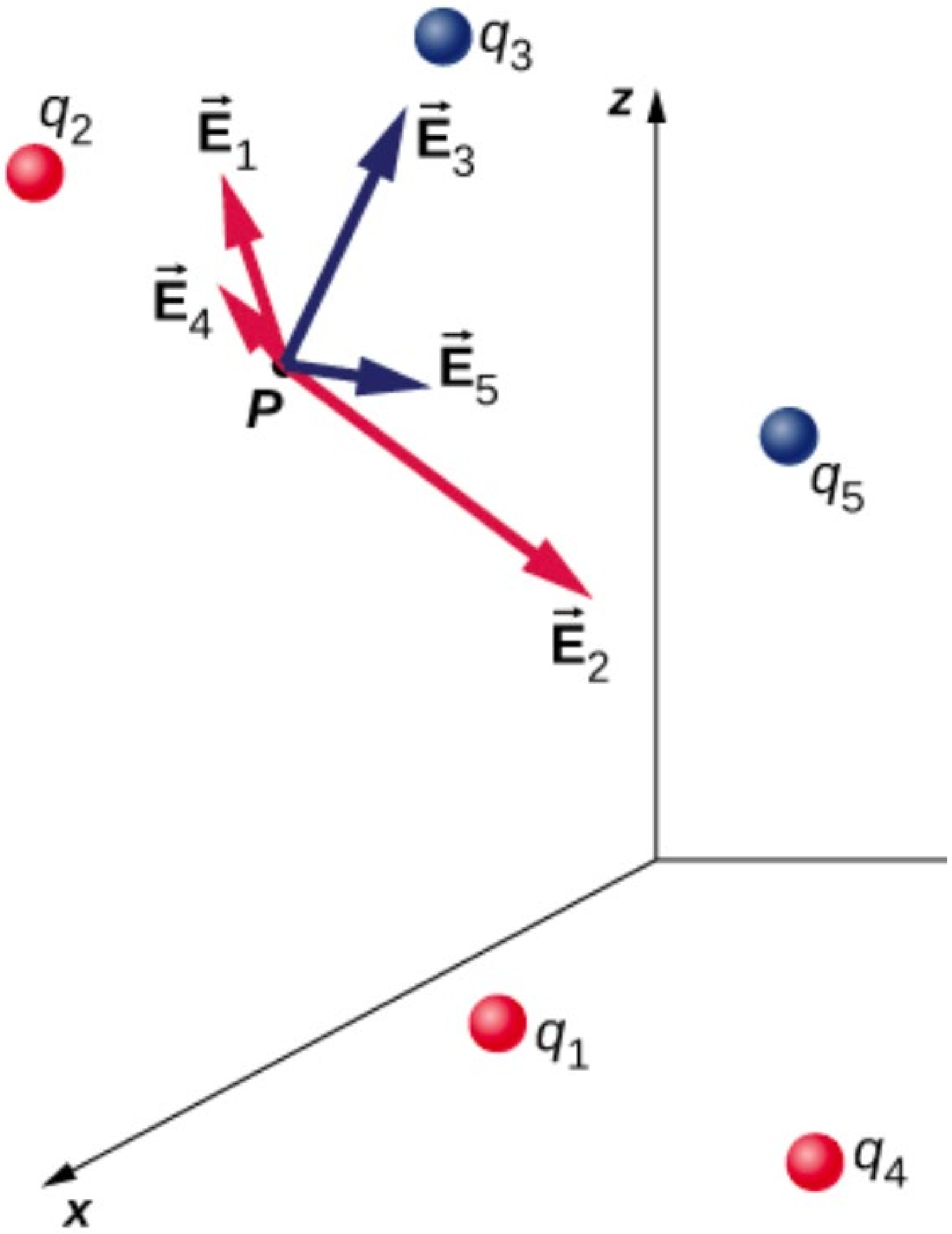
5.5: Electric Field

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_N$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Qq_1}{r_1^2}\hat{r}_1 + \frac{Qq_2}{r_2^2}\hat{r}_2 + \frac{Qq_3}{r_3^2}\hat{r}_3 + \ldots + \frac{Qq_N}{r_1^2}\hat{r}_N\right)$$

$$= Q\left[\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots + \frac{q_N}{r_1^2}\hat{r}_N\right)\right]$$

$$\vec{F} = Q\vec{E}$$



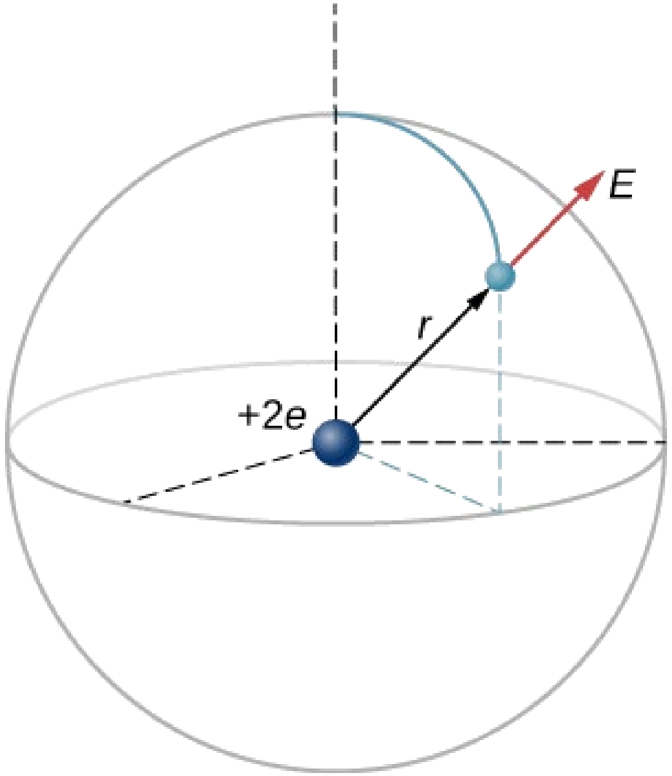
Electric Field $\vec{E}(\vec{r})$ or $\vec{E}(\boldsymbol{P})$

$$\vec{E} \equiv \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots + \frac{q_N}{r_1^2}\hat{r}_N\right)$$

$$\vec{E}(P) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i.$$

Direction of the Field

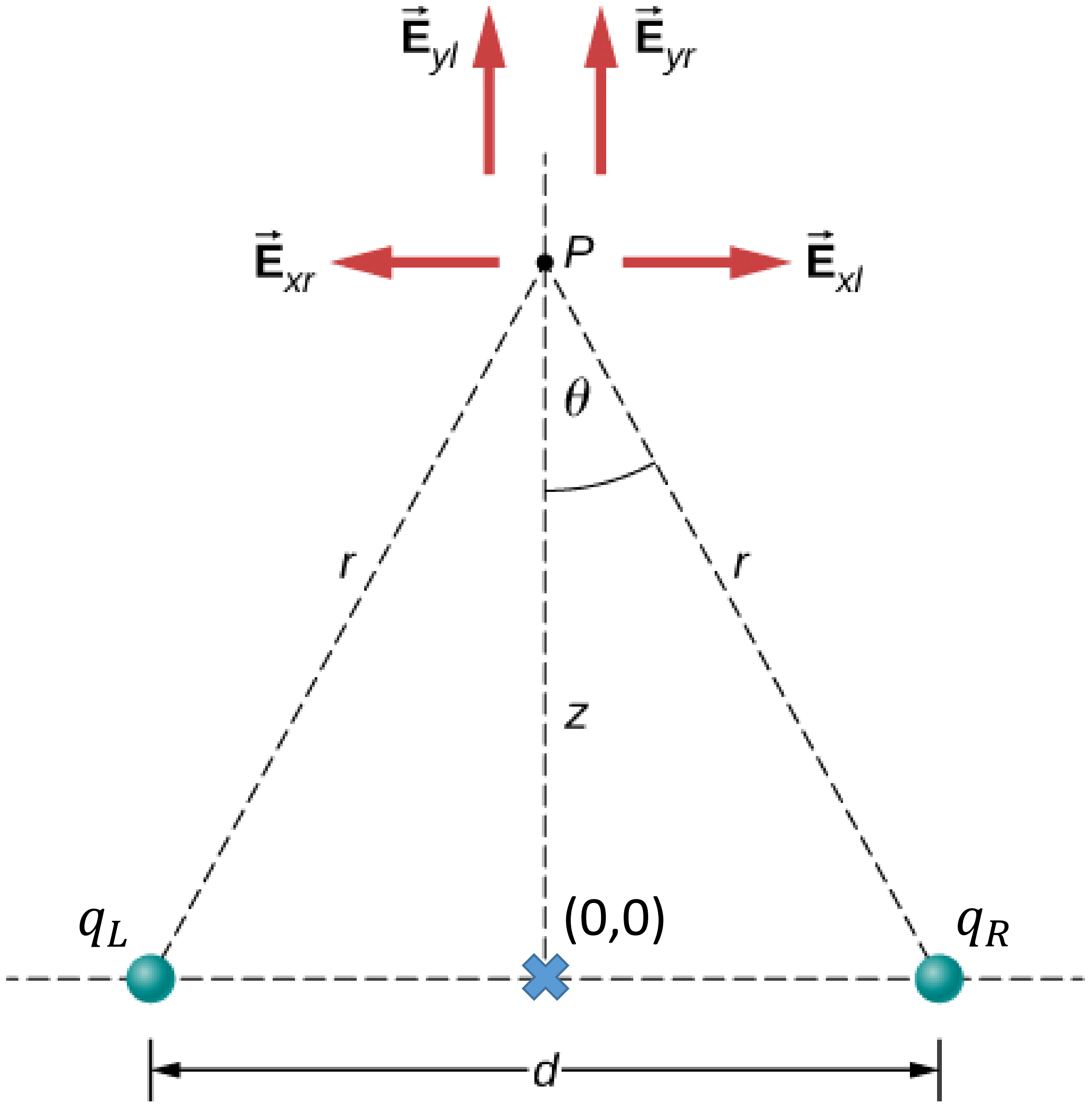
In an ionized helium atom, the most probable distance between the nucleus and the electron is $r = 26.5 \times 10^{-12}$m .What is the electric field due to the nucleus at the location of the electron?



$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i$$

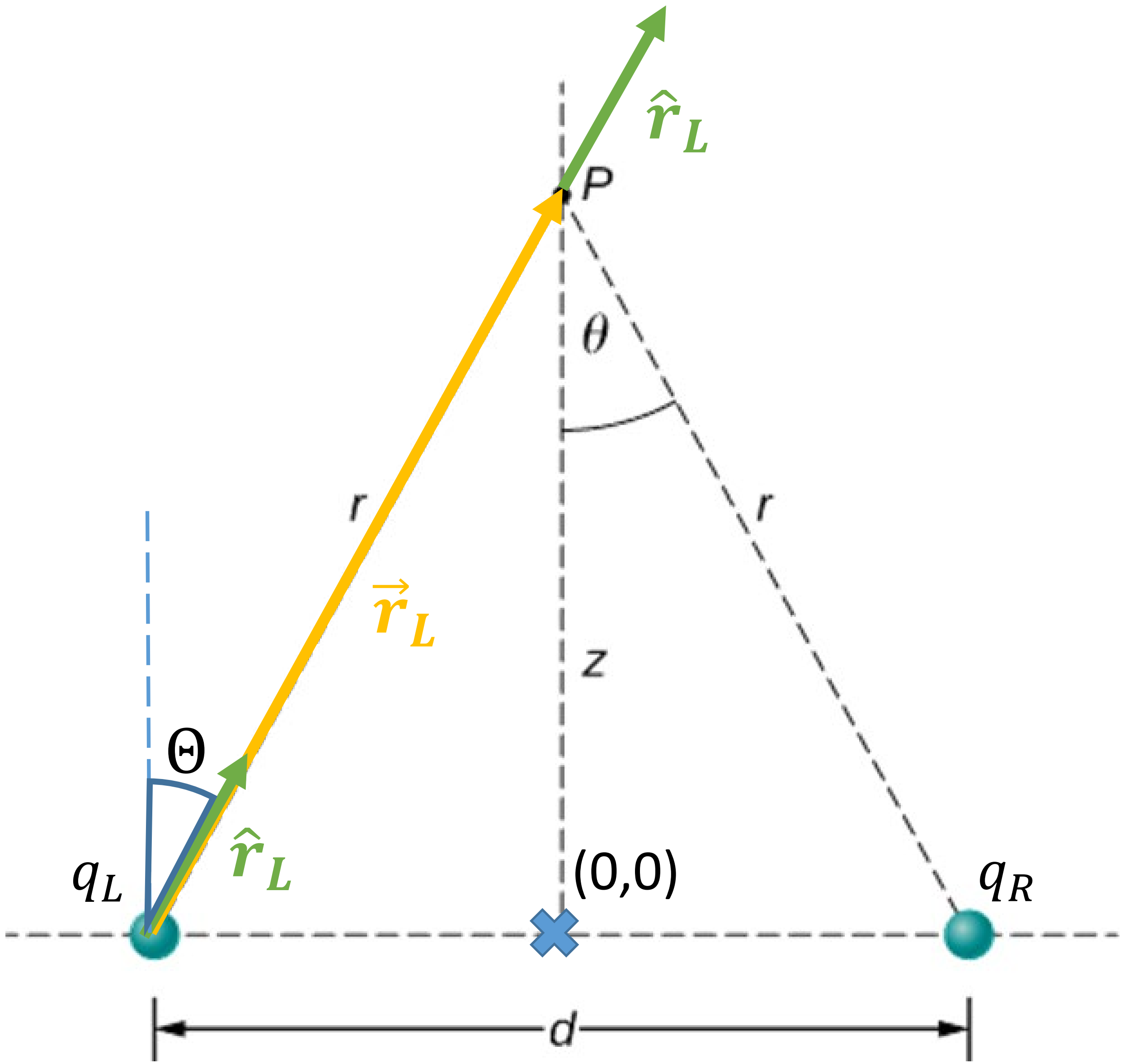
The E-Field above Two Equal Charges

- Vector notation of $\vec{E}_L(\vec{P})$, with direction vectors $\hat{r}_L$
- Vector notation of $\vec{E}_R(\vec{P})$, with direction vectors $\hat{r}_R$

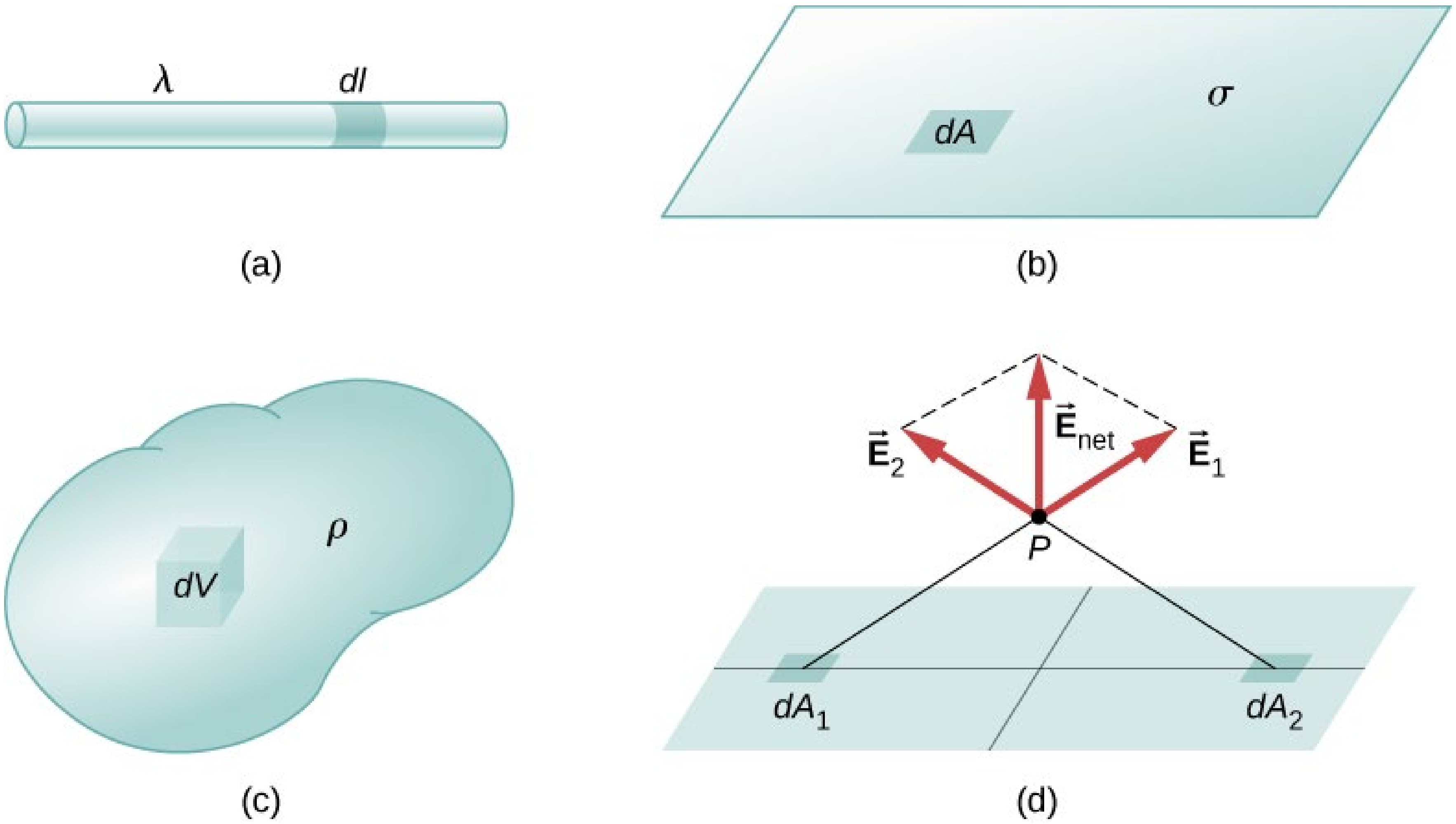


The E-Field above Two Equal Charges

- Vector notation of $\vec{E}_L(\vec{P})$, with direction vectors $\hat{r}_L$
- Vector notation of $\vec{E}_R(\vec{P})$, with direction vectors $\hat{r}_R$



5.6: Calculating Electric Fields of Charge Distributions



Charge Densities

- **linear** charge density:
  λ ≡ charge per unit length,
  units: coulombs **per meter** (C/m)
- **surface** charge density:
  σ ≡ charge per unit area,
  units: coulombs **per square meter** (C/m^2)
- **volume** charge density:
  ρ ≡ charge per unit volume,
  units: coulombs **per cubic meter** (C/m^3)

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \left(\frac{q_i}{r^2}\right) \hat{r}}_{\text{Point charges}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0} \int_{line} \left(\frac{\lambda\, dl}{r^2}\right) \hat{r}}_{\text{Line charge}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0} \int_{surface} \left(\frac{\sigma\, dA}{r^2}\right) \hat{r}}_{\text{Surface charge}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0} \int_{volume} \left(\frac{\rho\, dV}{r^2}\right) \hat{r}}_{\text{Volume charge}}$$

Charge Densities

- **linear** charge density:
  λ ≡ charge per unit length,
  units: coulombs per meter (C/m)

Solving these integrals is challenging.
Analytical solutions only for special cases,
but interesting cases!
Maybe later …

volume charge density:
ρ ≡ charge per unit volume,
units: coulombs **per cubic meter** (C/m^3)

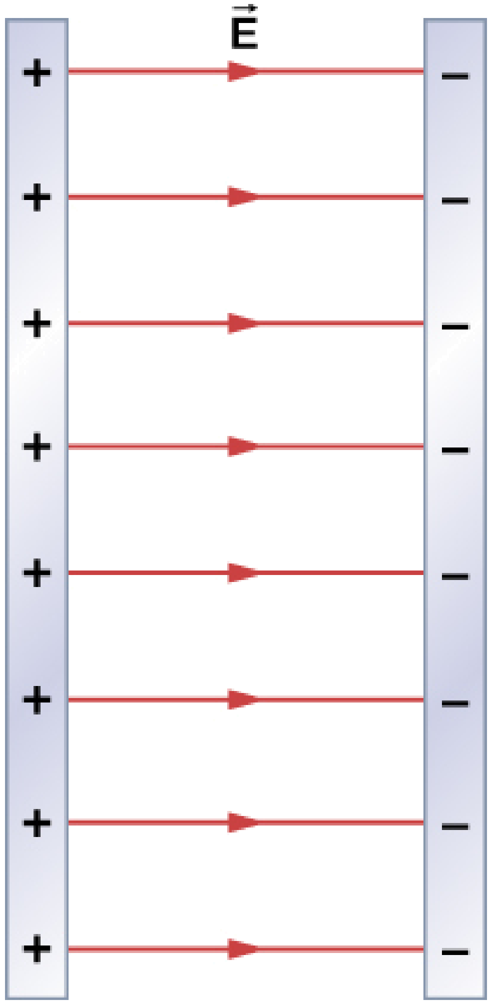
$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\left(\frac{q_i}{r^2}\right)\hat{r}}_{\text{Point charges}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\int_{line}\left(\frac{\lambda\, dl}{r^2}\right)\hat{r}}_{\text{Line charge}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\int_{surface}\left(\frac{\sigma\, dA}{r^2}\right)\hat{r}}_{\text{Surface charge}}$$

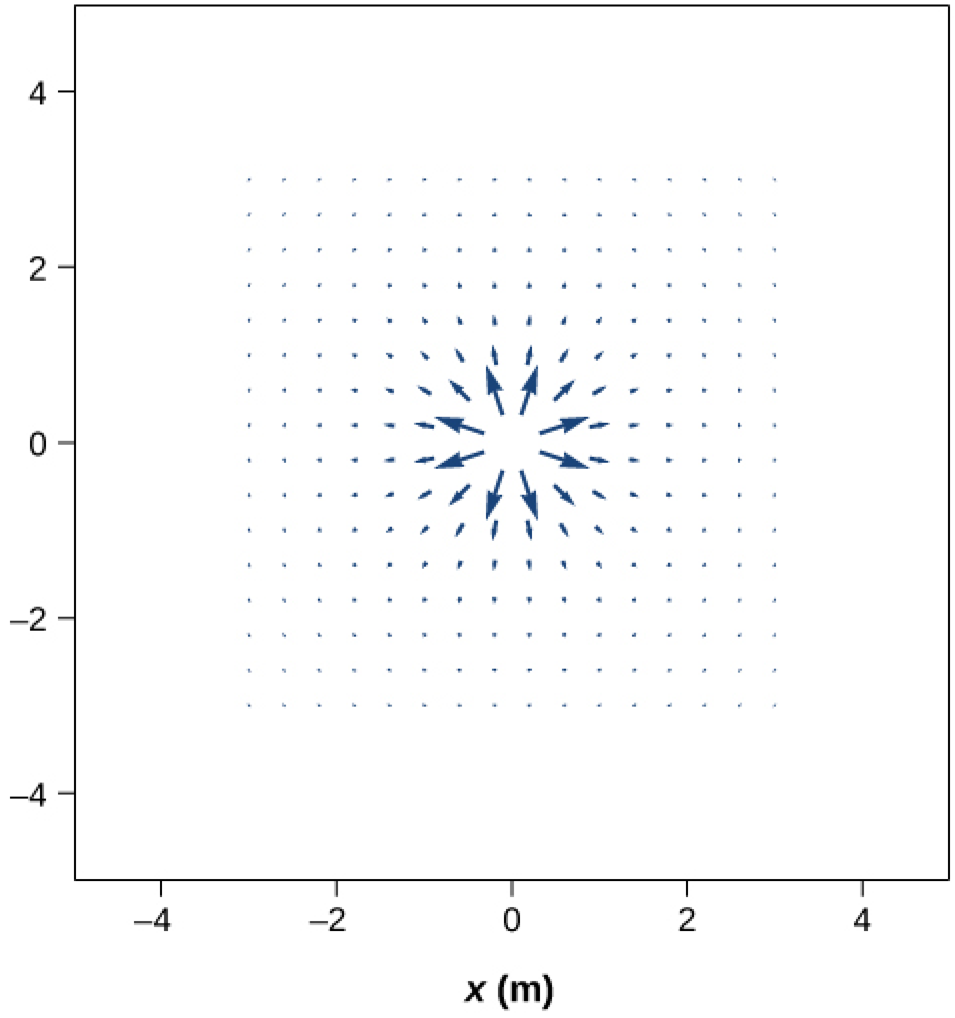
$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\int_{volume}\left(\frac{\rho\, dV}{r^2}\right)\hat{r}}_{\text{Volume charge}}$$

The Field of Two Infinite Planes: Important! Capacitor!

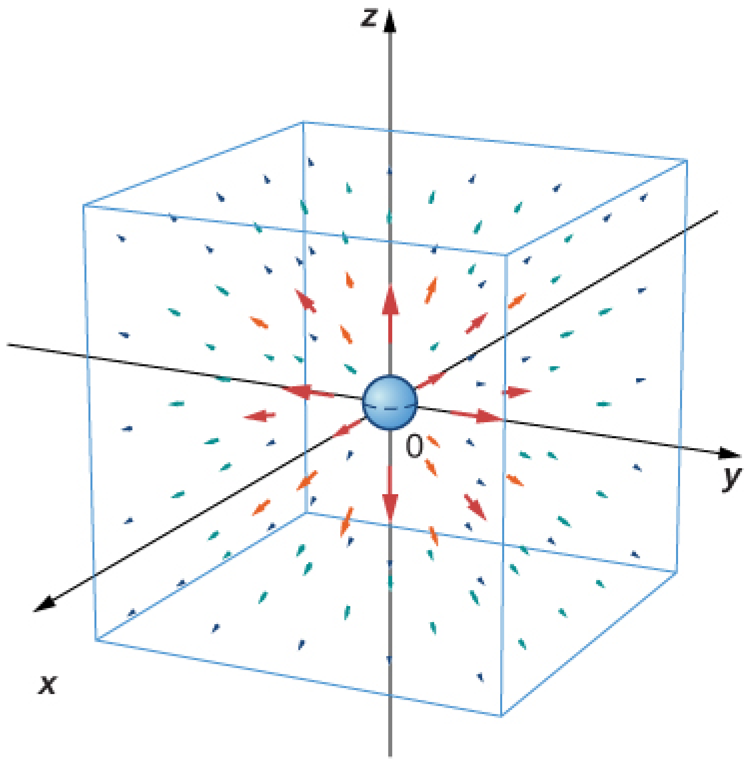


$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{i}$$

5.7: Electric Field Lines



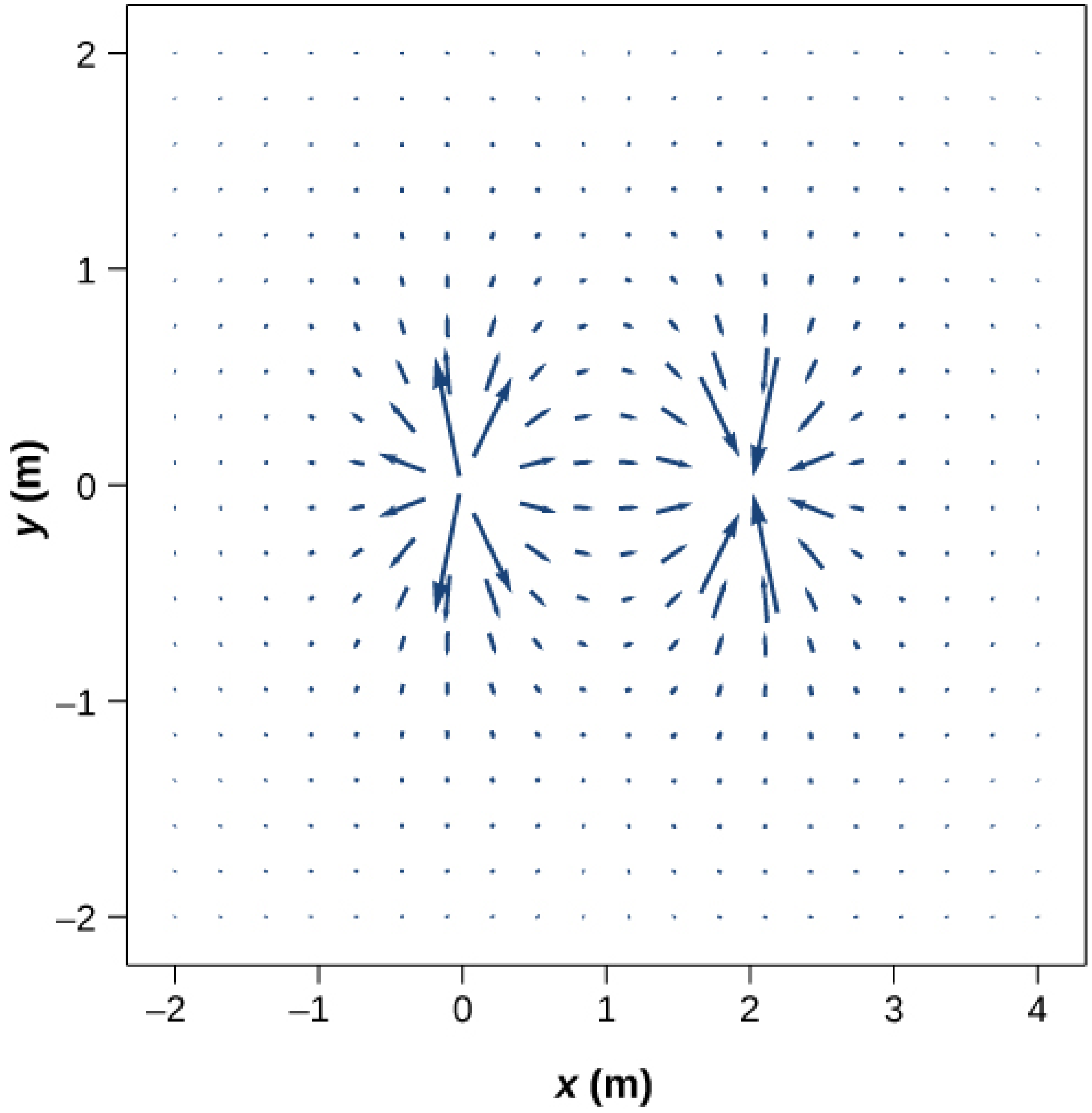
(a)



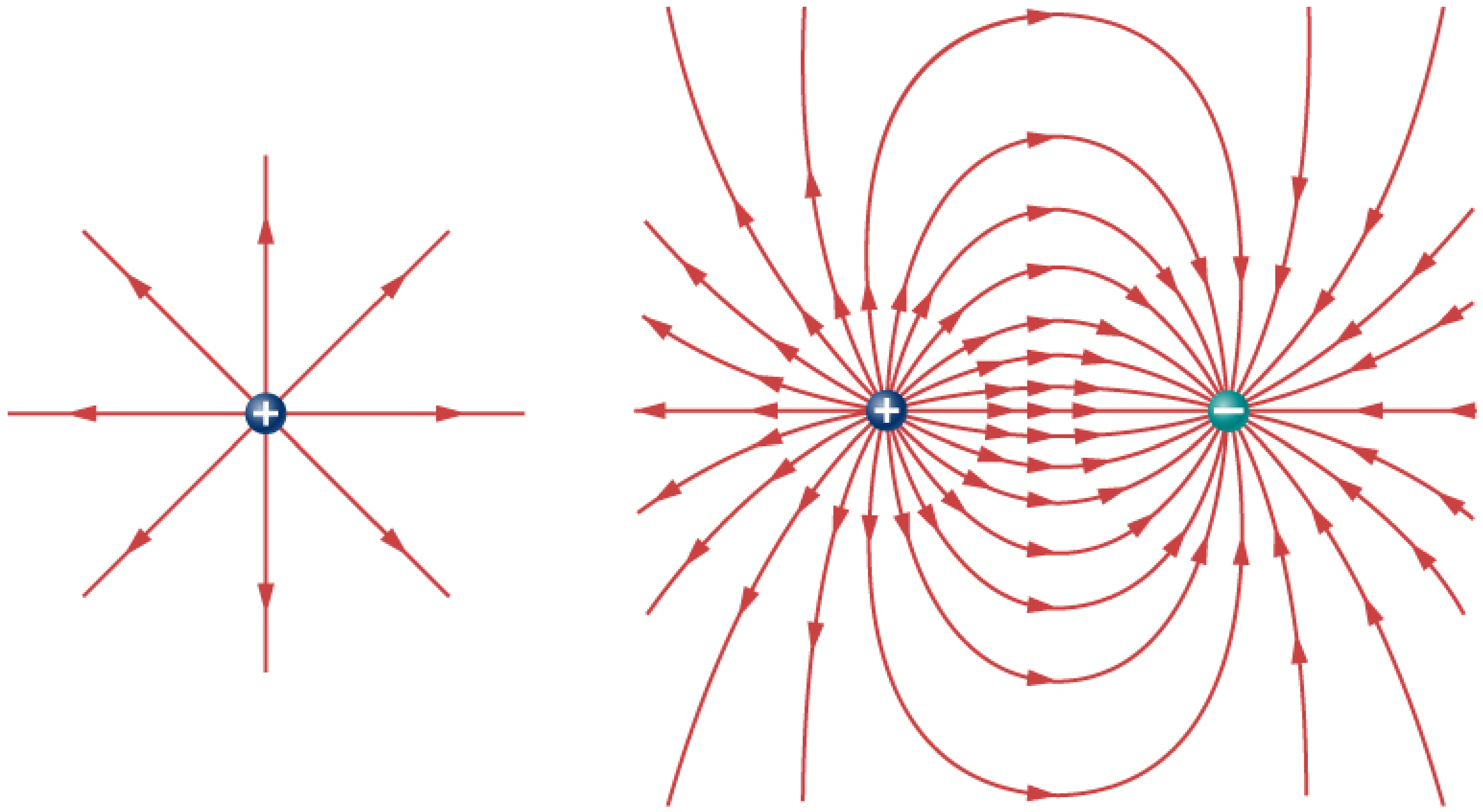
(b)

Vector Field of a Dipole

- Vector length is field strength
- Vector direction is direction of E-Field
- Attention: Diagram “distorted”. Axis scaling different.

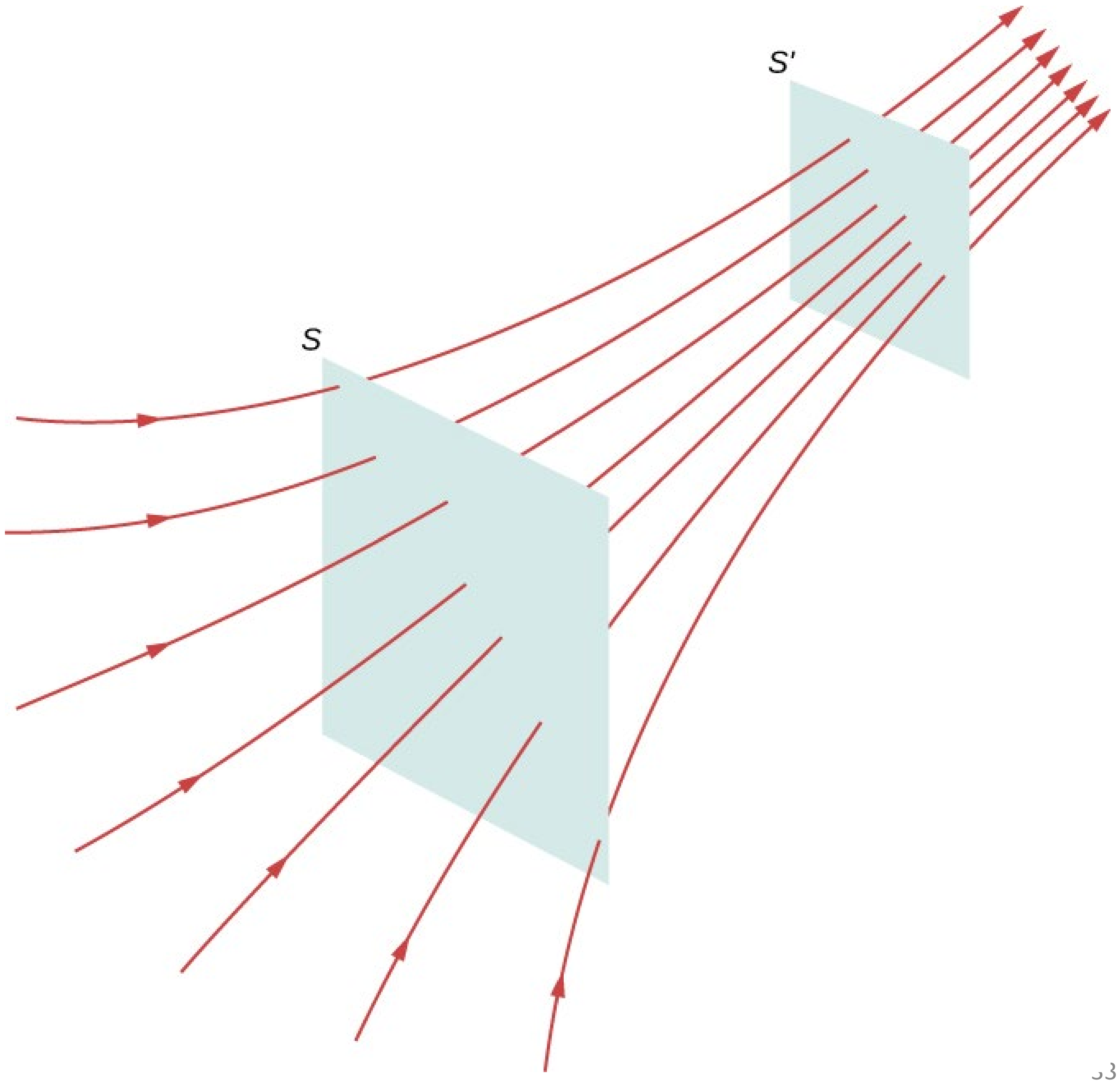


Field Lines as Continuous Lines

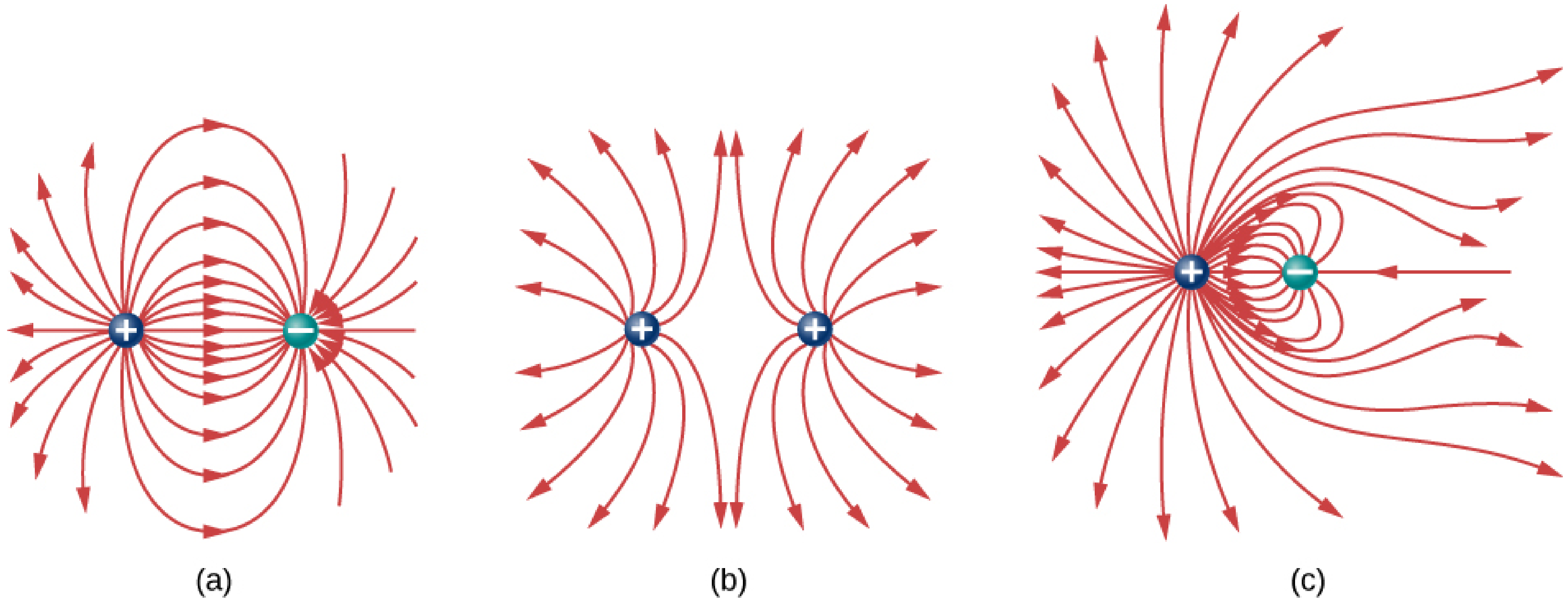


(a) (b)

Field Density: Flux



Three typical electric field diagrams.

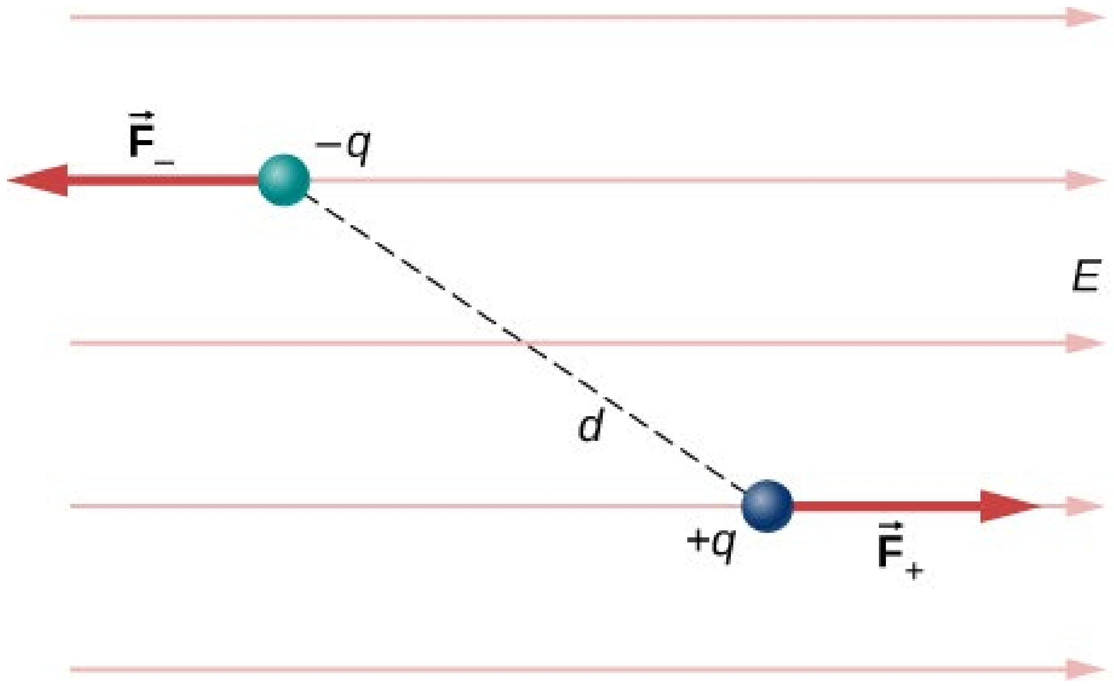


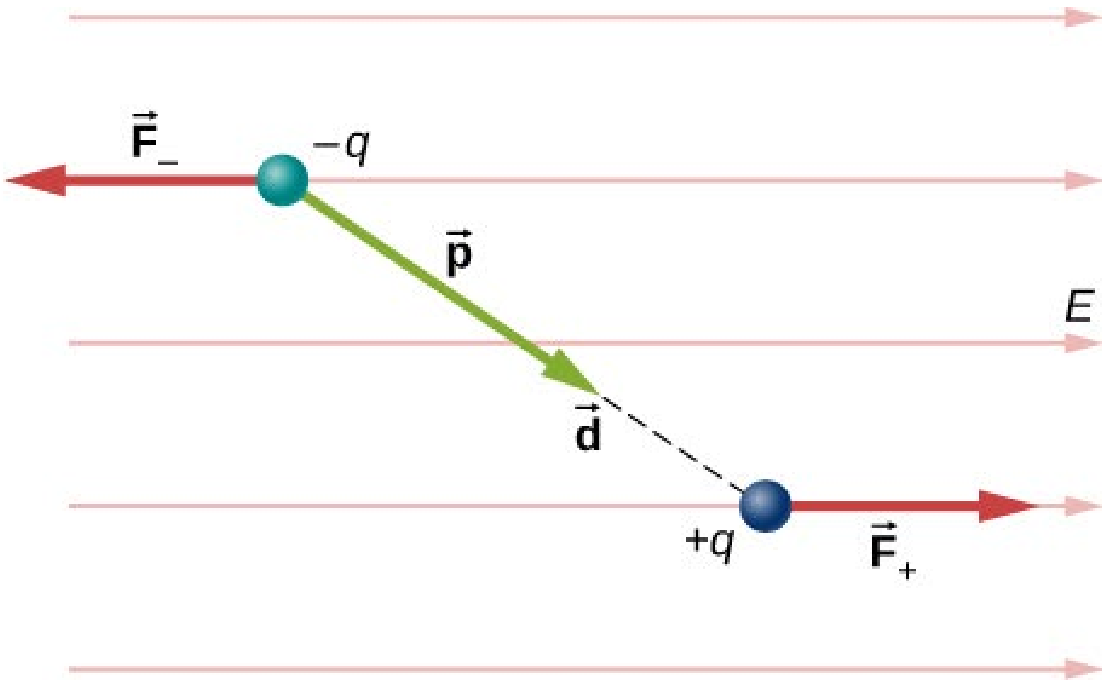
(a) A dipole. (b) Two identical charges. (c) Two charges with opposite signs and different magnitudes. Can you tell from the diagram which charge has the larger magnitude?

INTERACTIVE!

5.8: Electric Dipoles: Rotation of a Dipole due to an Electric Field

- Fixed distance between charges.
- Coulomb Forces are exerted. Leeds to torque.





Torque

$$\vec{\tau} = \left(\frac{\vec{d}}{2} \times \vec{F}_+\right) + \left(-\frac{\vec{d}}{2} \times \vec{F}_-\right)$$

$$= \left[\left(\frac{\vec{d}}{2}\right) \times \left(+q\vec{E}\right) + \left(-\frac{\vec{d}}{2}\right) \times \left(-q\vec{E}\right)\right]$$

$$= q\vec{d} \times \vec{E}. = \vec{p} \times \vec{E}$$