Chapter 24 – Capacitance and Dielectrics

- Capacitors and capacitance

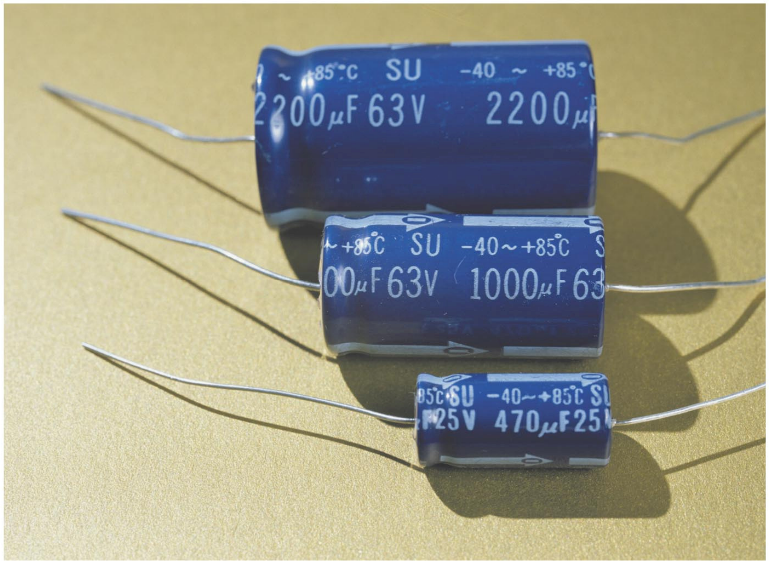
- Capacitors in series and parallel

- Energy storage in capacitors and electric field energy

- Dielectrics

- Molecular model of induced charge

- Gauss law in dielectrics



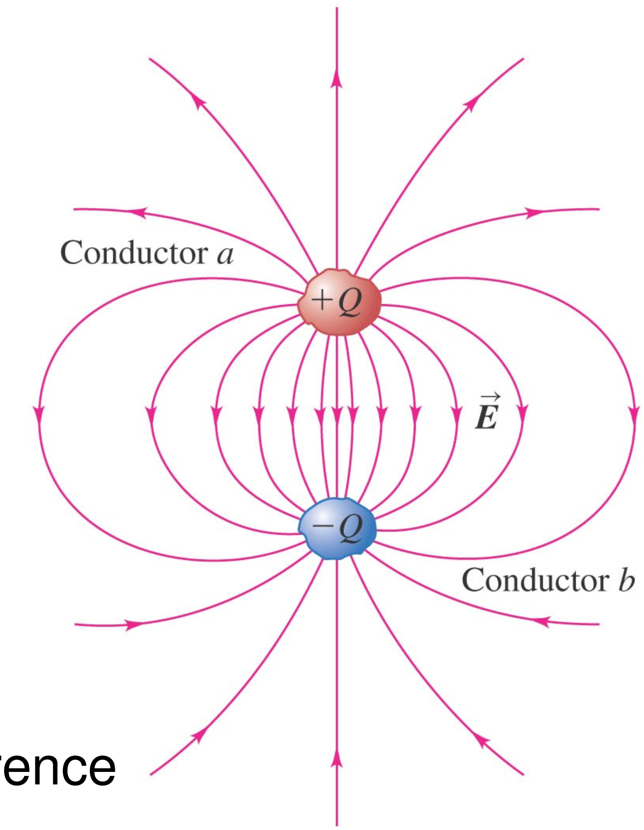
1. Capacitors and Capacitance

Capacitor: device that stores electric potential energy and electric charge.

- Two conductors separated by an insulator form a capacitor.

- The net charge on a capacitor is zero.

- To charge a capacitor -| |-, wires are connected to the opposite sides of a battery. The battery is disconnected once the charges Q and –Q are established on the conductors. This gives a fixed potential difference V_{ab} = voltage of battery.



Capacitance: constant equal to the ratio of the charge on each conductor to the potential difference between them.

$$\boxed{C = \frac{Q}{V_{ab}}}$$

Units: 1 Farad (F) = Q/V = C^2/J = C^2/N m

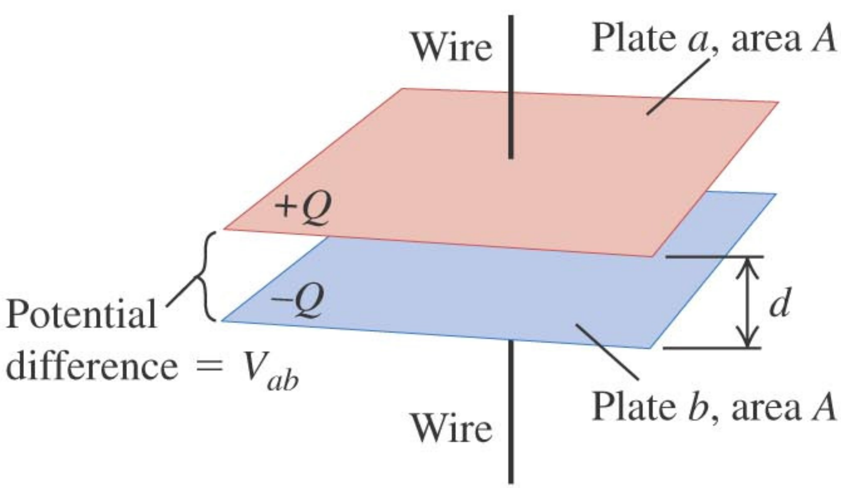
- Capacitance is a measurement of the ability of capacitor to store energy (V = U / q).

Capacitors in Vacuum

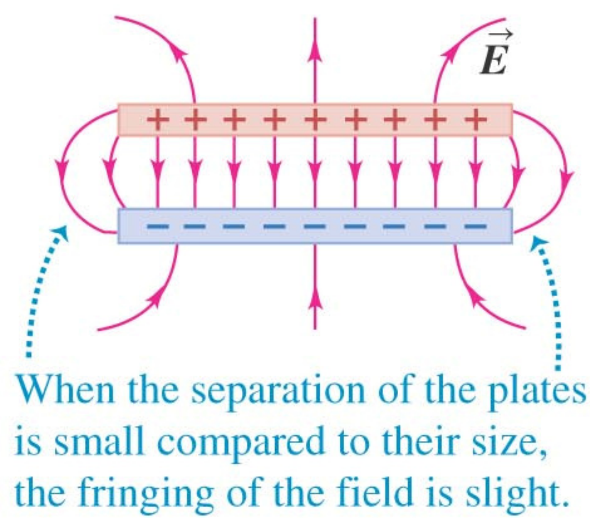
- Parallel Plate Capacitor: uniform electric field between the plates, charge uniformly distributed over opposite surfaces

$$E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{\varepsilon_o A} \qquad V_{ab} = E \cdot d = \frac{1}{\varepsilon_o}\frac{Qd}{A} \qquad \boxed{C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d}}$$

(a) Arrangement of the capacitor plates



(b) Side view of the electric field $\vec{E}$



ε_0 = 8.85 x 10^{-12} F/m

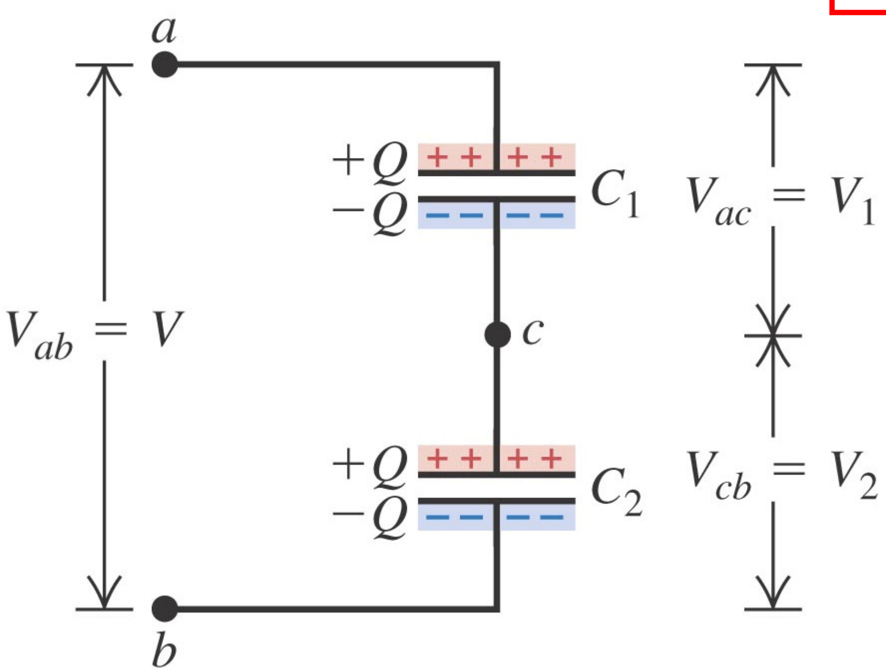
- The capacitance depends only on the geometry of the capacitor.

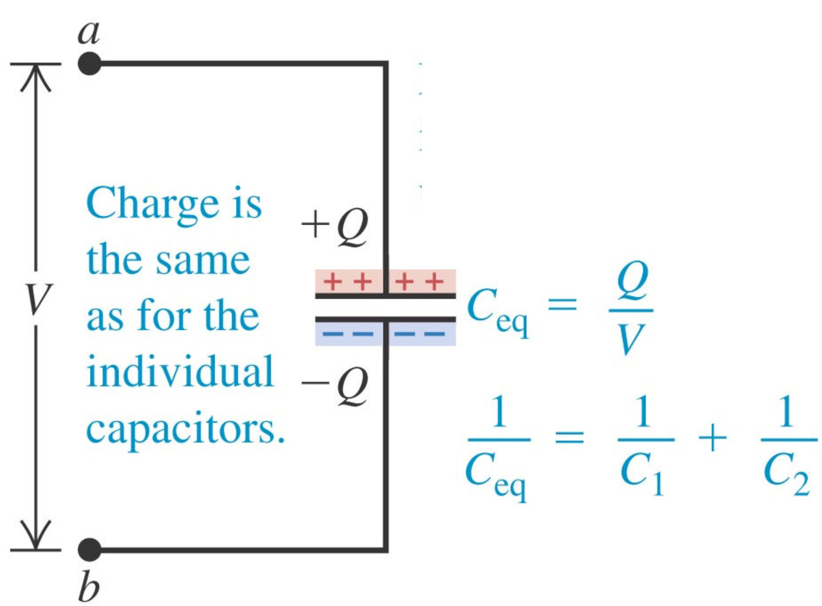
2. Capacitors in Series and Parallel

Capacitors in Series:

- Same charge (Q). $\boxed{V_{ab} = V_{ac} + V_{cb}}$

$$C_{eq} = \frac{Q}{V_{ab}} = \frac{Q}{V_1 + V_2} \quad \rightarrow \quad \boxed{\frac{1}{C_{eq}} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}}$$





Equivalent capacitor

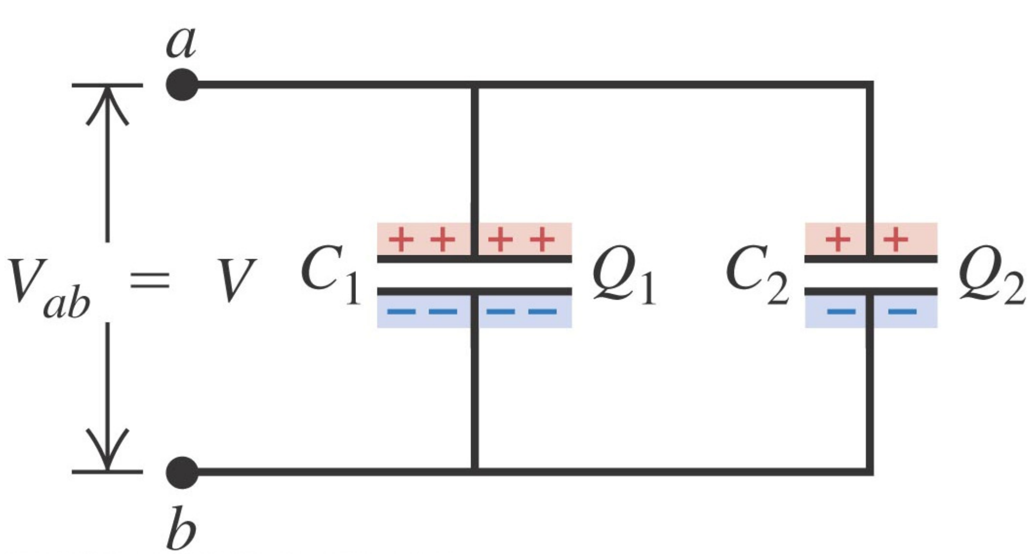
Capacitors in Parallel:

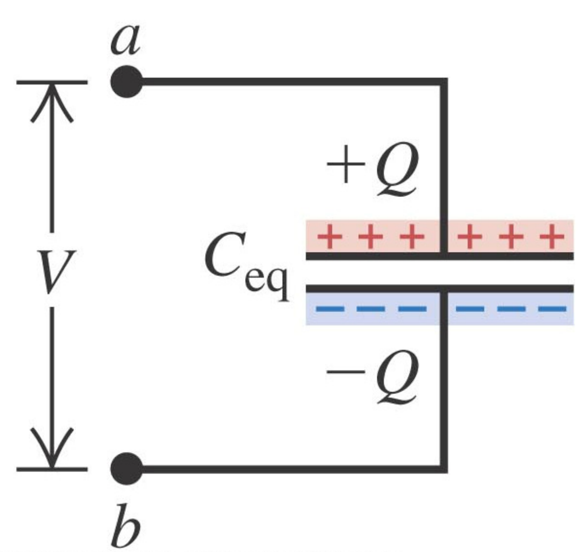
- Same potential V, different charge.

$\boxed{Q_1 = C_1 V_1}$ $\boxed{Q_2 = C_2 V_2}$

$\boxed{Q = Q_1 + Q_2}$

$$C_{eq} = \frac{Q}{V_{ab}} = \frac{Q_1 + Q_2}{V} \quad \rightarrow \quad \boxed{C_{eq} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2}$$





Equivalent capacitor

3. Energy Stored in Capacitors and Electric-Field Energy

- The electric potential energy stored in a charged capacitor is equal to the amount of work required to charge it.

Work to charge a capacitor:

$$dW = dU = v \cdot dq = \frac{q \cdot dq}{C}$$

$$\boxed{W = \int_0^W dW = \frac{1}{C}\int_0^Q q \cdot dq = \frac{Q^2}{2C}}$$

- Work done by the electric field on the charge when the capacitor discharges.

- If U = 0 for uncharged capacitor → W = U of charged capacitor

Potential energy stored in a capacitor:

$$\boxed{U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}}$$

Electric-Field Energy:

- A capacitor is charged by moving electrons from one plate to another. This requires doing work against the electric field between the plates.

Energy density: energy per unit volume stored in the space between the plates of a parallel-plate capacitor.

$$\boxed{u = \frac{\frac{1}{2}CV^2}{A \cdot d}} \qquad C = \frac{\varepsilon_0 A}{d} \qquad V = E \cdot d$$

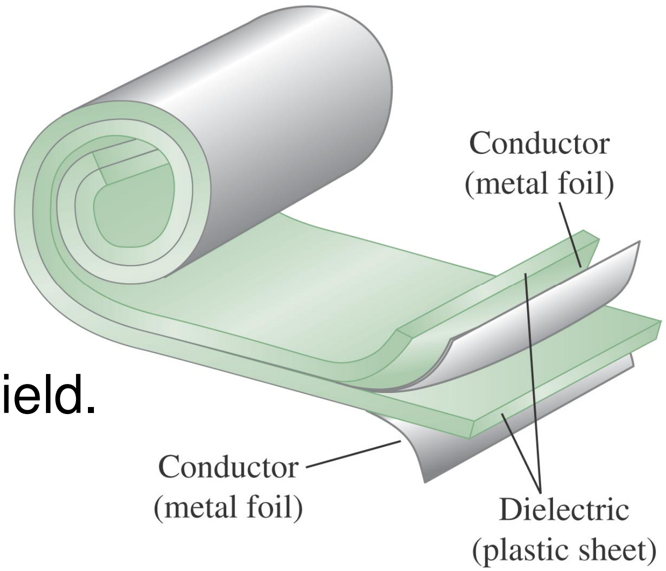
Electric Energy Density (vacuum):

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

4. Dielectrics

- Non-conducting materials between the plates of a capacitor. They change the potential difference between the plates of the capacitor.

-The dielectric layer increases the maximum potential difference between the plates of a capacitor and allows to store more Q.

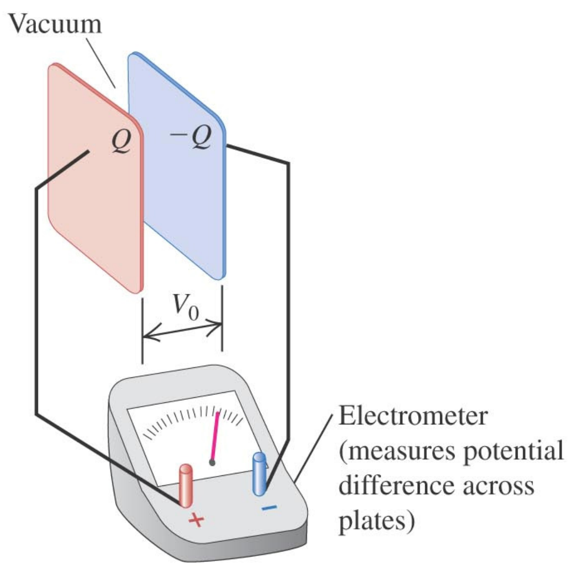


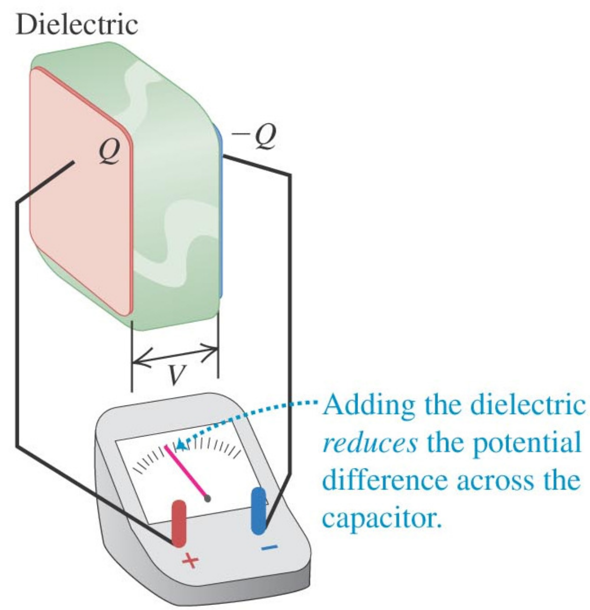
Dielectric breakdown: partial ionization of an insulating material subjected to a large electric field.

Dielectric constant (K): $$\boxed{K = \frac{C}{C_0}}$$

C = capacitance with the dielectric inside the plates of the capacitor

C_0 = capacitance with vacuum between the plates





- If Q = constant → $Q = C_0 V_0 = C V$ → $C/C_0 = V_0/V$

$$\boxed{V = \frac{V_0}{K}}$$

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

- No real dielectric is a perfect insulator → always leakage current between charged plates of a capacitor with a dielectric.

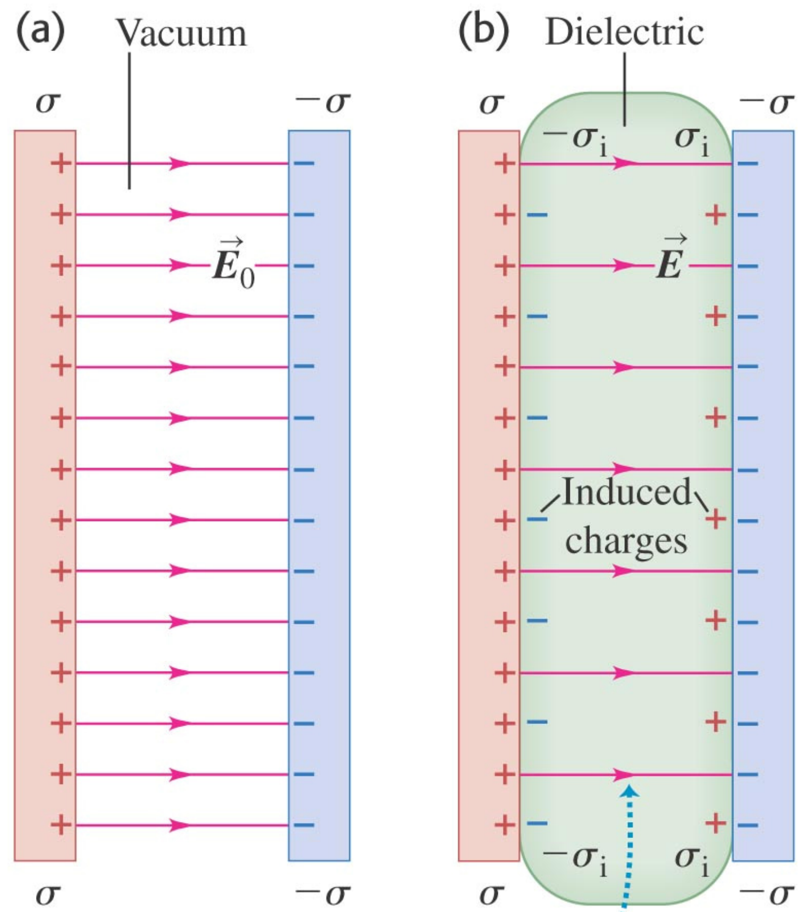
Induced Charge and Polarization:

$$\boxed{E = \frac{E_0}{K}}$$ (Q constant)

E = field with the dielectric between plates
E_0 = field with vacuum between the plates

- E is smaller when the dielectric is present →surface charge density smaller. The surface charge on conducting plates does not change, but an induced charge of opposite sign appears on each surface of the dielectric. The dielectric remains electrically neutral (only charge redistribution).

Polarization: redistribution of charge within a dielectric.



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

Field lines change in the presence of dielectrics.

-The induced surface density in the dielectric of a capacitor is directly proportional to the electric field magnitude in the material.

Net charge on capacitor plates: (σ-σ_i) (with σ_i = induced surface charge density)

$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad E = \frac{E_0}{K} = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

Induced surface charge density: $\boxed{\sigma_i = \sigma\left(1 - \frac{1}{K}\right)}$

Permittivity of the dielectric: $\boxed{\varepsilon = K\varepsilon_0}$

Electric field (dielectric present): $E = \dfrac{\sigma}{\varepsilon}$

Capacitance of parallel plate capacitor (dielectric present): $\boxed{C = K \cdot C_0 = K\varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}}$

Electric energy density (dielectric present): $\boxed{u = \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2}\varepsilon \cdot E^2}$

Dielectric breakdown:

A very strong electrical field can exceed the strength of the dielectric to contain it.

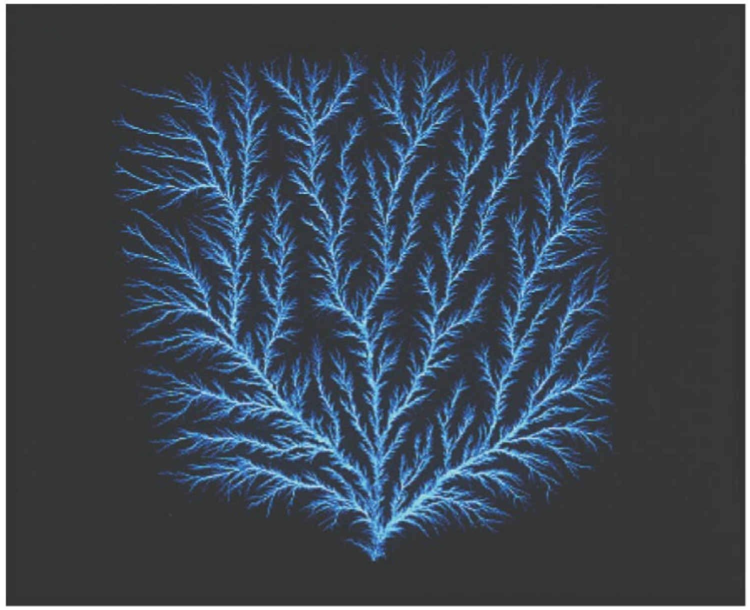
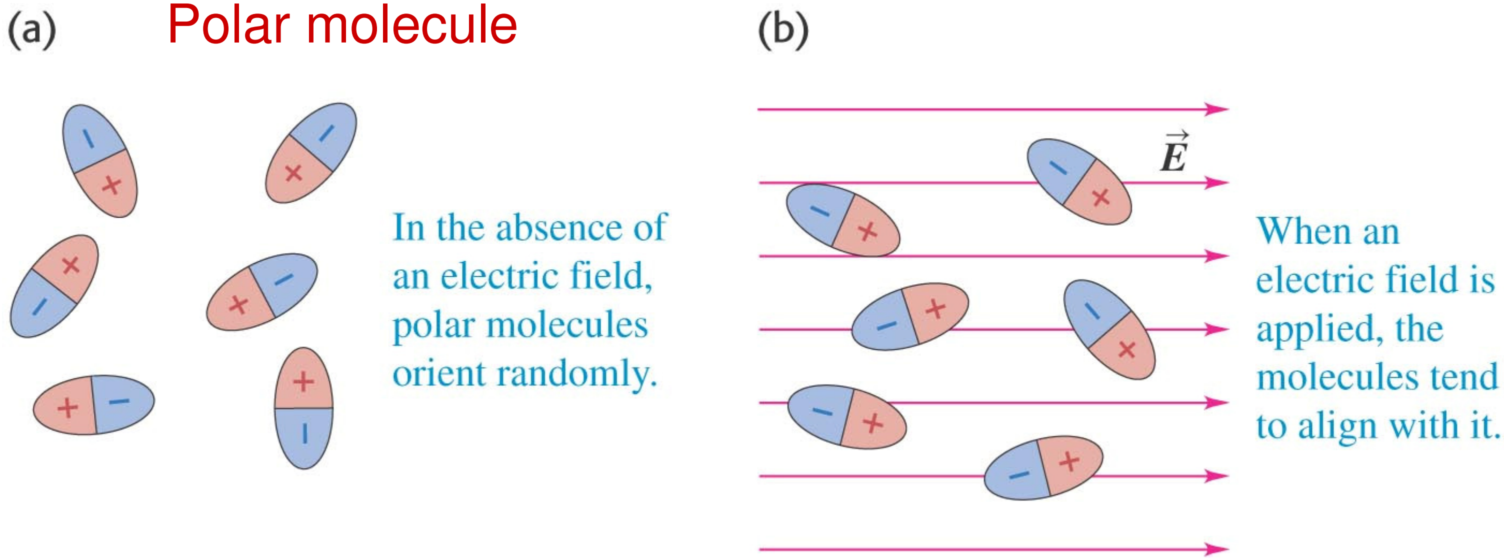


Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

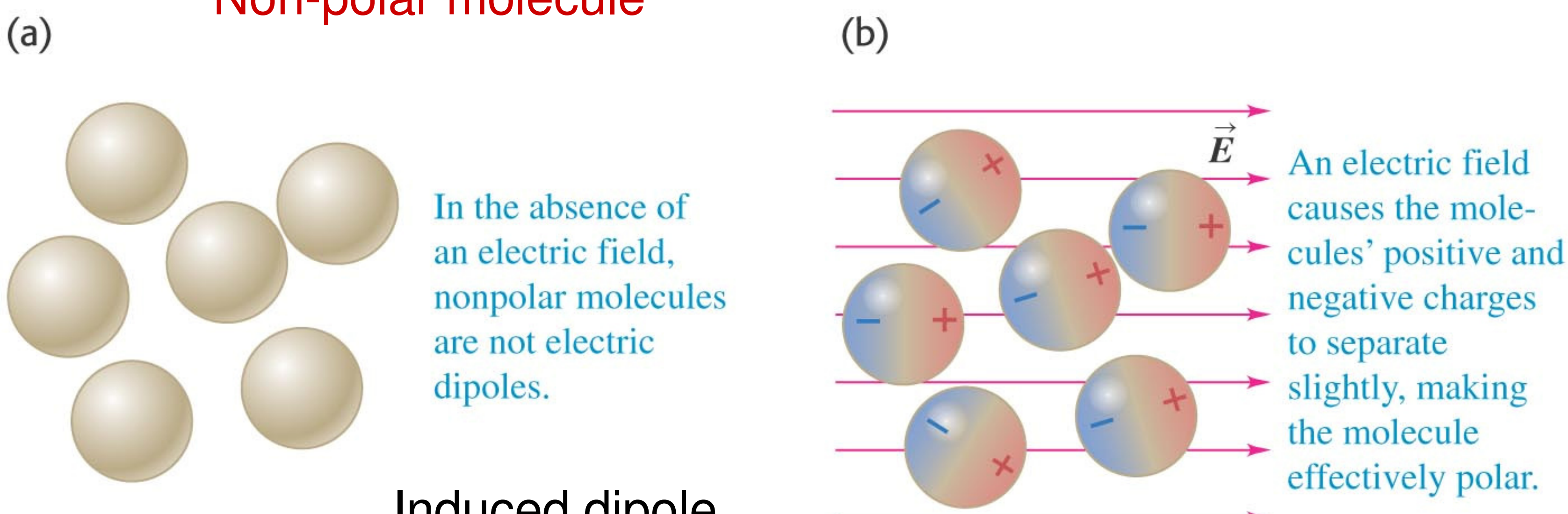
Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

5. Molecular Model of Induced Charge

Polar molecule

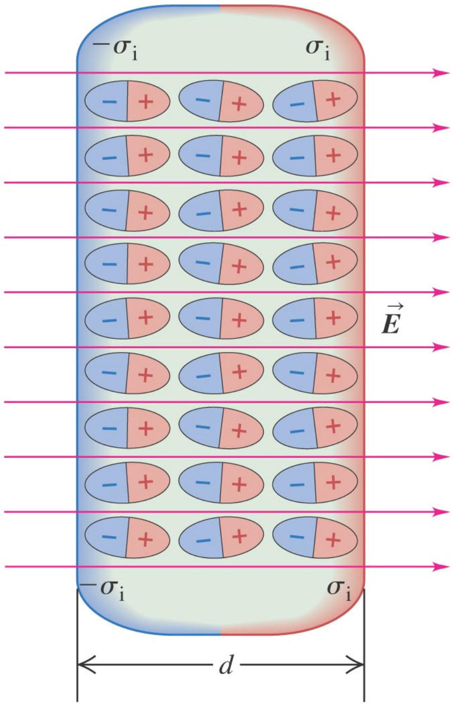


Non-polar molecule

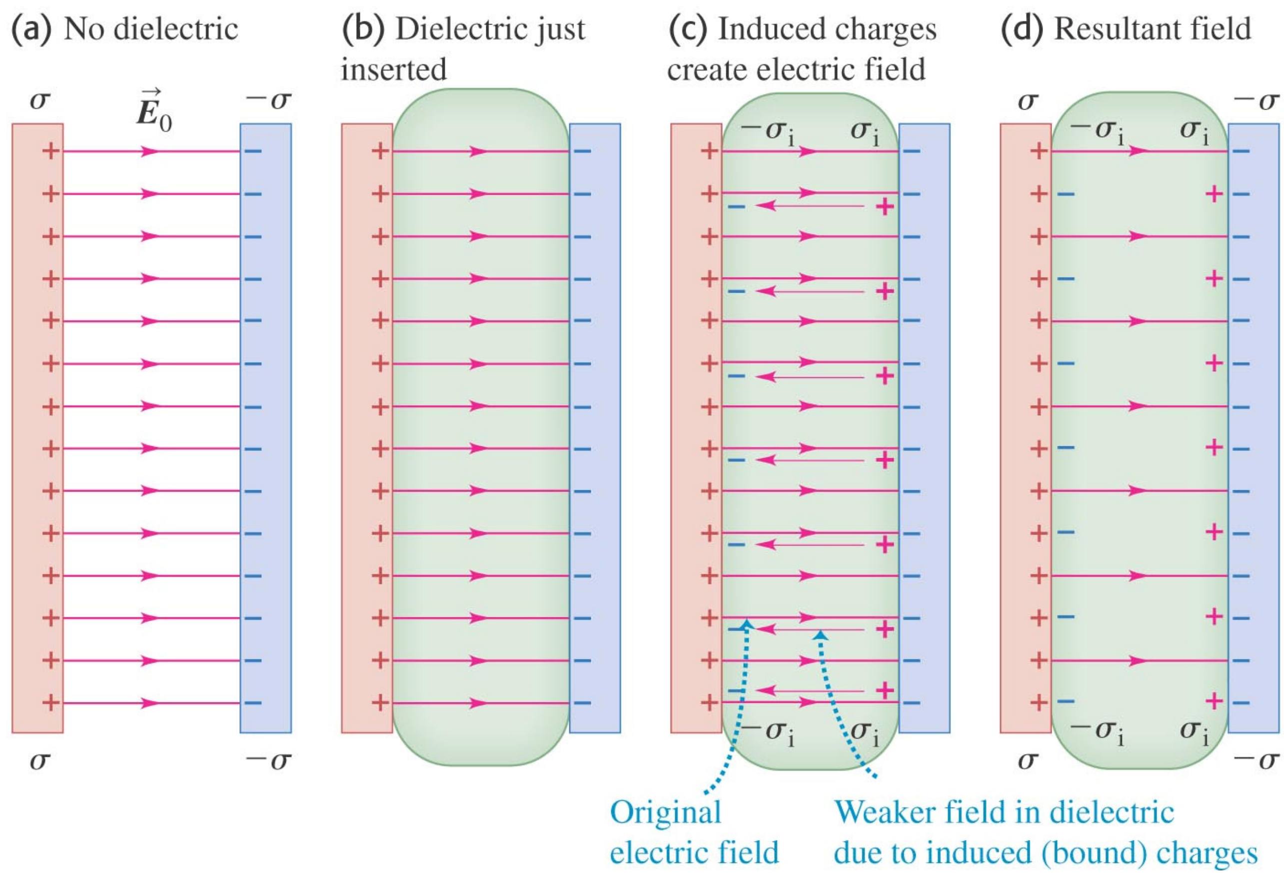


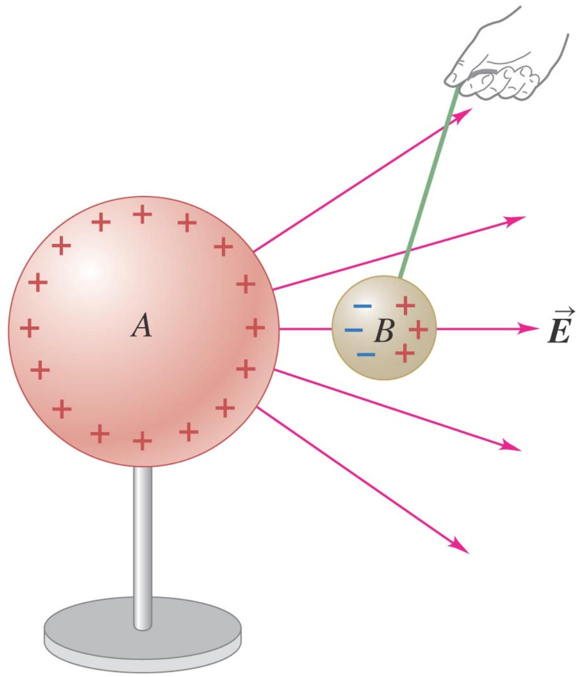
Induced dipole

Polarization and Electric Field Lines

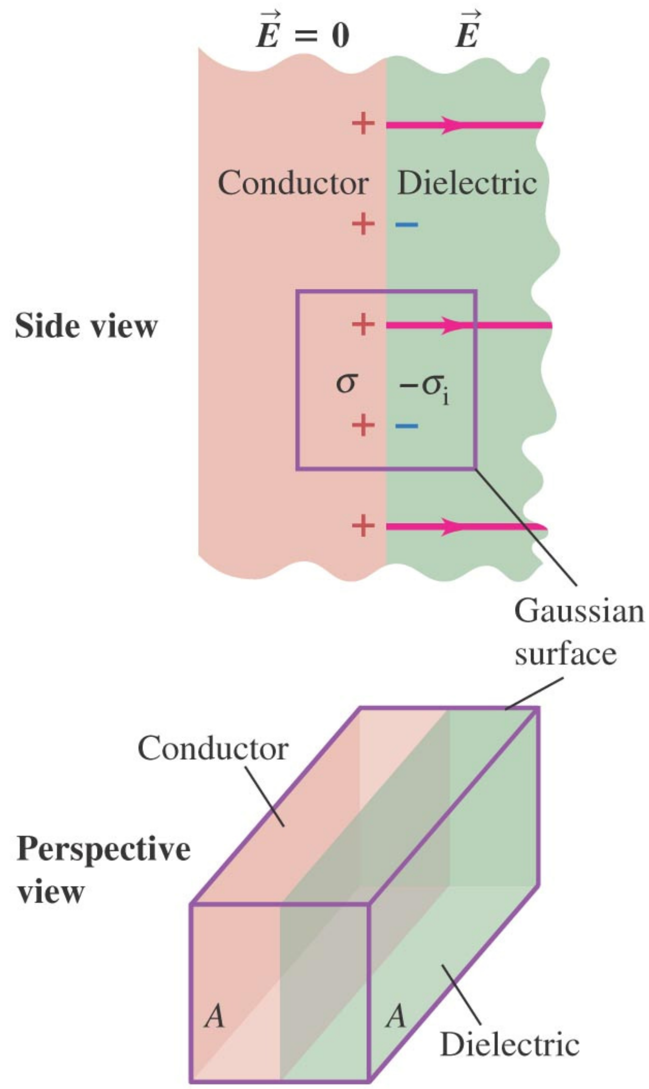


Polarization of a dielectric in electric field gives rise to bound charges on the surfaces, creating σ_i, - σ_i.





A neutral sphere B in the radial electric field of a positively charged sphere A is attracted to the charge because of polarization.



6. Gauss's Law in Dielectrics

$$EA = \frac{Q_{encl}}{\varepsilon_0} = \frac{(\sigma - \sigma_i)A}{\varepsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

$$EA = \frac{\sigma \cdot A}{K\varepsilon_0}$$

Flux through Gaussian surface (enclosed free charge / ε_0)

$$KEA = \frac{\sigma \cdot A}{\varepsilon_0}$$

Gauss Law in a dielectric:

$$\boxed{\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{encl-free}}{\varepsilon_0}}$$